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AXLE LOAD EQUALISATION IN MECHANICAL TRAILER SUSPENSIONS

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CONTENTS

PREFACE
by Transport and Road Research Laboratory

Multi-axle bogies of heavy goods vehicle semi-trailers are required to share the load on the bogie equally between axles. Roadside surveys of the axle weights of heavy goods vehicles have shown that semi-trailer bogies fitted with steel leaf springs often have axle weights that are not equal (Mitchell, 1987). Laboratory tests show that when a semi-trailer is tipped nose up the rear axle of its bogie overloads and the front axle unloads. The physical processes that give rise to these experimental and survey observations are not fully understood in detail.

In 1983 TRRL commissioned consultants, Smallfry Ltd, to conduct a study of existing HGV suspension systems to show how they purport to work, how they could be expected to work in practice and the expected advantages and disadvantages of each system. Smallfry Ltd reported in July 1985 (Smallfry, 1985). This report concluded:

1. No mechanical equalisation system which relies on balance arms or beams to equalise loads between closely spaced axles can achieve consistent equalisation in practice if more than one balance arm is pivoted on the vehicle chassis.

2. Even mechanical load equalisation systems which rely on a single pivot on the chassis will not be able to equalise load exactly:

- if any factor such as tyre wear or spring hysteresis causes geometric change in the system without changing load

- if there is sufficient rotational stiffness in any pivot in the equalisation mechanism.

3. Theoretical analysis, even to a first degree of approximation, shows that the axle load-distribution inequalities, measured within the triple-axle group on vehicles in service using the TRRL dynamic weighbridge, are to be expected in practice while the vehicle is in motion. They are largely the result of interaction between trailer chassis tilt and the suspension.

4. The causes of these inequalities of the load-distribution are such that they represent the steady-state axle loads.

5. The system can only equalise if the trailer chassis and the line through the centres of the axles are both exactly parallel to the road surface. Any factor which causes a change in this relationship will cause a maldistribution of load as long as the factor lasts. Permanent disturbances, such as a tilt in the trailer chassis, will cause permanent maldistribution.

Smallfry (1985) included a finite element analysis of a 3-axle bogie suspension which had been carried out by sub-contractors, ETA Engineering Consultants Ltd. In 1986 TRRL placed a contract with ETA to study the static and dynamic behaviour of a 3-axle leaf spring HGV trailer suspension. The report that follows, by ETA Engineering Consultants Ltd, is a summary of the results obtained by this contract. ETA concluded that a mechanical equalisation system using balance arms should give good equalisation, even for 3-axle bogies, if the equaliser bushes had zero torsional stiffness. In the event, the results of the mathematical analysis did not agree well with experimental results obtained in the laboratory, but did agree better with experimental results obtained by another organisation. The physical requirements for good equalisation systems are not yet established, and it is considered that the report should be published because no other analyses of bogie equalisation systems are known in the literature. It is hoped that the analysis presented in the following report will be a useful contribution to future studies of these mechanisms.

PREFACE REFERENCES

C G B Mitchell (1987) The effect of the design of goods vehicle suspensions on loads on roads and bridges Department of Transport, TRRL Research Report RR115, Transport and Road Research Laboratory, Crowthorne.

Smallfry Ltd (1985) Axle load compensation in tandem and tri-axle suspensions TRRL Working Paper VED/85/15, Vehicles and Environment Division, Transport and Road Research Laboratory, Crowthorne.

PROLOGUE

The work described in this report is a sequence of investigations whose aim has been to fully understand why mechanically equalised multi-axle trailer suspensions are ineffective when tested under roadside conditions. The approach has been to study theoretically the mechanics of such suspensions to find out what inherent design features there might be which lead to lack of equalisation.

The project fell into four distinct stages under the following headings:-

Stage 1. A Specific Finite Element Analysis.

Stage 2. An Algebraic Linear Static Model.

Stage 3. A Dynamic Linear Model.

Stage 4. A Dynamic Non-linear Analysis.

All four stages have been described fully in previous reports, (ref 1 to 4), but the main features of each are summarised in this present document, so as to present an overall view of the study.

Stage 1. A Specific Finite Element Analysis.

This was a preliminary foray in which a particular tri-axle suspension was modelled using beam finite elements and spring elements, with pin joints where necessary, to achieve representative kinematics. The loading condition was a translation and rotation applied to the trailer bed.

Various runs were made with differences in tyre and spring properties in order to see whether variations of these parameters could explain the experimentally observed lack of equalisation.

The analysis confirmed that even an idealised linear model of the suspension was unable to equalise axle loads, but did not yield any positive indication as to the reason. It did show that the finite element model was an adequate means of representing a specific example of a mechanical suspension. On the other hand a very large number of analyses might be necessary to establish the relative importance of various design details by a parametric study of such a model.

Stage 2. An Algebraic Linear Static Model

Still retaining the simplifying assumption of linear theory, the next part of the study used an algebraic model, rather than a finite element model of the statically loaded suspension. This allowed general expressions to be derived for axle loads in terms of stiffnesses, lengths and trailer bed slope. The algebra was couched in non-dimensional terms so that it was applicable to any example of the generic type of suspension.

The static response of the load equalisation mechanism may be established by considering equilibrium of forces and moments along with the stiffnesses of the various components. The essential components of the mechanism are represented as shown in Figure 1 where points A,B,C and D are connected to the vehicle chassis frame.

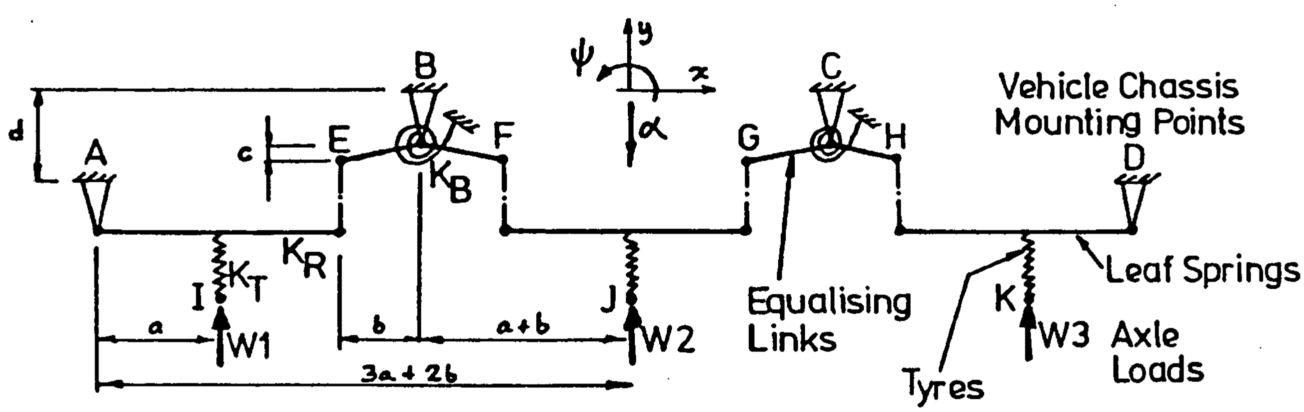

Figure 1

After some algebraic manipulation the following equation can be derived relating, (in a non-dimensional form), wheel loads to chassis displacement and rotation:-

$$\begin{bmatrix} 1+q & -q & 0 \\ -q & 1+2q & -q \\ 0 & -q & 1+q \end{bmatrix} \begin{bmatrix} \overline{W}_1 \\ \overline{W}_2 \\ \overline{W}_3 \end{bmatrix} = \begin{bmatrix} 1+d \\ 1 \\ 1-d \end{bmatrix} \qquad \text{———————— (1)}$$

where:-

q = $b^2K_s/4K_B$ stiffness ratio

K_S = spring stiffness of tyre and road spring ($^1/K_S = {}^1/K_T + {}^1/K_R$)

K_B = rotational stiffness of equaliser pivot bush

$\overline{W}_i$ = $W_i/\alpha K_s$; (i=1,2,3) non dimensional wheel loads

d = $\frac{\psi}{\alpha}(2a + 3b/2)$ generalised chassis rotation

α = downward displacement of chassis at centre of suspension

ψ = rotation of chassis

Solving these equations gives the non-dimensional wheel loads acting on the suspension system. The effectiveness of the system can be measured in terms of a wheel load deviation parameter

$$p = \sqrt{(\overline{W}_1 - \overline{W}_2)^2 + (\overline{W}_3 - \overline{W}_2)^2} \qquad \text{———————— (2)}$$

which will be zero if the loads $\overline{W}_1$, $\overline{W}_2$ and $\overline{W}_3$ are equalised.

Figure 2 shows a family of curves derived from Equations 1 and 2. This clearly shows that equalisation of wheel loads is ineffective unless a relatively flexible equaliser pivot bush is used. Also indicated on the graph (by means of * symbols) are five points taken from the Stage 1 work.

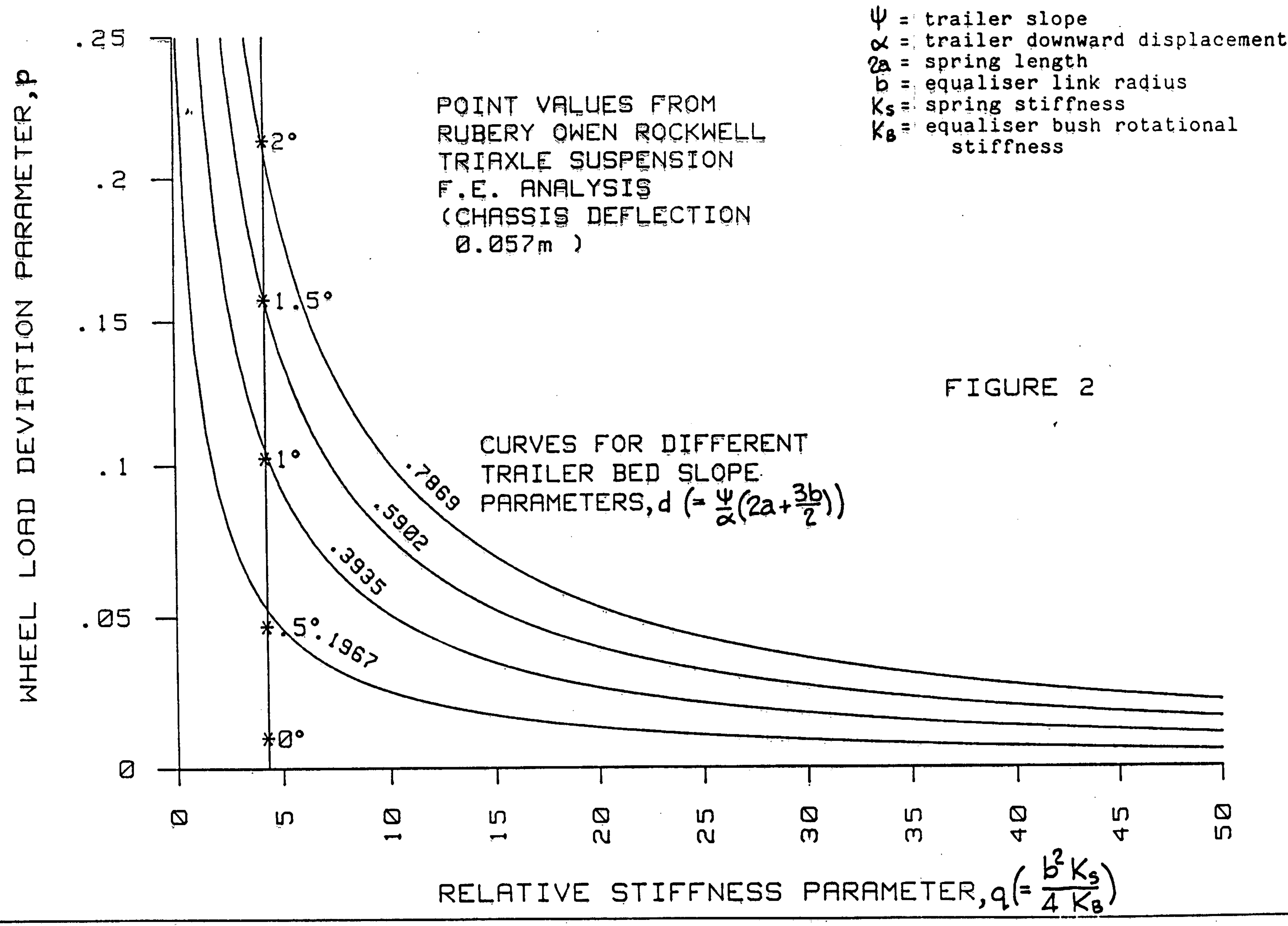
ψ = trailer slope
α = trailer downward displacement
2a = spring length
b = equaliser link radius
K_s = spring stiffness
K_B = equaliser bush rotational stiffness
POINT VALUES FROM RUBERY OWEN ROCKWELL TRIAXLE SUSPENSION F.E. ANALYSIS (CHASSIS DEFLECTION 0.057m)
CURVES FOR DIFFERENT TRAILER BED SLOPE PARAMETERS, d $\left(= \frac{\psi}{\alpha}\left(2a + \frac{3b}{2}\right)\right)$
.7869
.5902
.3935
.1967
2°
1.5°
1°
.5°
0°
WHEEL LOAD DEVIATION PARAMETER, p
.25
.2
.15
.1
.05
0
RELATIVE STIFFNESS PARAMETER, q $\left(= \frac{b^2 K_s}{4 K_B}\right)$
0
5
10
15
20
25
30
35
40
45
50

FIGURE 2

These show good agreement with the Stage 2 algebraic approach, although a small deviation is apparent for low angles of slope. The reason for this is a lack of symmetry, introduced in the finite element model by the drag links; a refinement which does not appear in the algebraic model.

Despite this minor discrepancy, the two analyses indicate clearly the same lack of equalisation. Moreover, the calculations confirm experimental information which suggests widespread occurrence of unequalised axle loads. Similar results were obtained for two-axle suspensions.

For the suspension modelled in Stage 1 the stiffness parameter was 4.3. This value is so low that effective equalisation of axle loads is impossible. The reason is that the relatively stiff bush absorbs most of the torque on the equaliser link resulting from the difference of adjacent spring forces; thus preventing the transmission of axle loads to adjacent springs. Applied to this particular suspension the curves in Figure 2 represent the range from zero slope up to a 2 degree chassis inclination in ½ degree steps, with a chassis downward displacement of 57 mm. The line for zero slope coincides with the (p = 0) axis.

The conclusion drawn at this stage was that, within the limitation of the assumption of linearity, a zero stiffness equaliser bush should give good equalisation.

Stage 3. Dynamic Linear Model

The previous work suggested that static performance would be improved by using equaliser bushes with little or no rotational stiffness. The next consideration was whether the choice of a freely rotating bush would also improve equalisation under dynamic conditions. Further, we needed to investigate whether some degree of stiffness may be necessary to the achievement of acceptable road holding.

For purposes of the dynamic investigation it was decided to examine the 2 axle case since not only was the theory much less complex than with three axles but also the results could be compared if necessary with those from an actual road vehicle under test at TRRL.

A model similar to the algebraic one was used in the dynamic approach, the suspension components being idealised in a finite element manner and the equations of motion derived so as to allow excitation of a step input from the road surface. The model included the front trailer king pin pivot, so that some coupling existed between the suspension displacement and trailer slope. The model used for the study is shown in Figure 3.

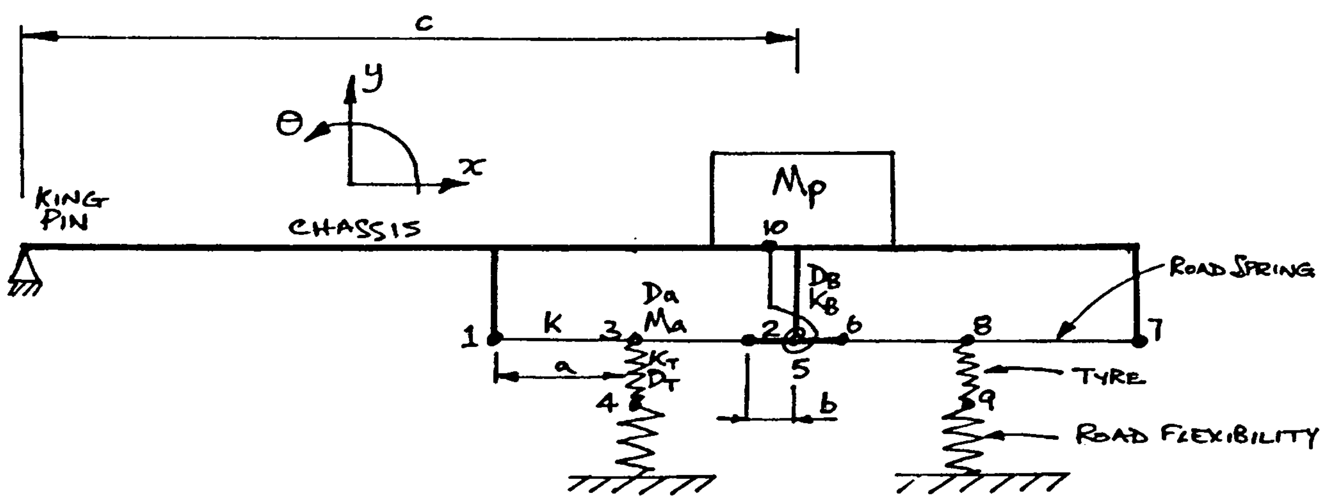

Figure 3

Equations of motion of the suspension system were expressed non-dimensionally in the form:-

$$[M]\ [\ddot{U}] + [C]\ [\dot{U}] + [K]\ [U] = [R] \qquad \text{———————————— (3)}$$

where [K] denotes the assembled stiffness matrix
[C] denotes the assembled damping matrix
and [M] denotes the assembled mass matrix

each of these being (6 x 6) matrices.

Full details regarding the computation of numerical values for the matrices is given in reference 3.

Because of the complexity of these equations of motion an analytic approach is impractical, but by using numerical integration, solutions can be computed for step inputs, (or for any other time varying input). The Newmark method was chosen for the present work, mainly because it has the advantage of unconditional stability. That is, the solution is stable regardless of time step size. On the other hand a suitable time step has to be chosen to achieve accuracy.

The Newmark solution was programmed for a Hewlett Packard desktop computer, and tested on a simple 2 degrees of freedom problem for which there was documented an analytic solution. Following satisfactory testing, this routine was used to solve the equations of motion in an appropriate computer environment which allowed the response history for any freedom in the system to be plotted. In addition to displacements, velocities and accelerations, this computer program also calculates the wheel loads and wheel parameter throughout the time history so that the degree of equalisation is immediately apparent.

Seven runs were carried out with various values for the suspension operating parameters and computational constants as follows:-

Kb	=	1000/10 N/m	(bush stiffness)
Db	=	0/2000 Ns/m	(bush damping)
Ma	=	0/200 Kg	(axle mass)
M	=	400/200	(No of iterations)
Dt	=	.0025/.0625/.125 etc.	(time interval)
Vel	=	0.2/10 m/s	(vehicle velocity)

Other parameters were fixed to represent geometry similar to the suspension previously analysed.

The following table indicates the conditions for each run, together with the peak value of the wheel load distribution parameter.

TABLE 1 LINEAR DYNAMIC ANALYSIS RUNS

Run No	Kb (N/m)	Db (Ns/m)	Ma (Kg)	M	Dt (s)	Vel (m/s)	Peak Wheel Parameter		
							(P)	(P1)	
L01	1000	0	0	400	.0625	.2	0.2	.11	Low speed
L02	1000	0	0	400	.0025	10	0.95	.46	High speed
L03	10	0	0	200	.125	.2	0.14	.08	Low speed
L04	10	0	0	200	.005	10	1.05	.50	High speed
L05	1000	2000	0	400	.0025	10	1.55	.75	High speed
L06	1000	2000	0	400	.0625	.2	1.8	.89	Low speed
L07	1000	2000	200	400	.0625	.2	1.7	.85	Low speed

Note that two values are given for the peak wheel parameter. The first is the parameter p of Equation 2. The second value is an alternative measure of wheel load variation defined as

$$P1 = \sqrt{\frac{\sum(\overline{W} - \overline{W}_m)^2}{n}} \qquad (4)$$

where $\overline{W}$ is a non-dimensional wheel load
$\overline{W}_m$ is the mean of the non-dimensional wheel loads on the various axles.
n is the number of axles.

This second parameter was introduced as a more appropriate wheel load parameter following discussions at a project review meeting.

The excitation used in these analyses was a step in the road surface as defined in the diagram below. A ramp was assumed at each edge of the step to achieve a realistic motion of the wheel centre.

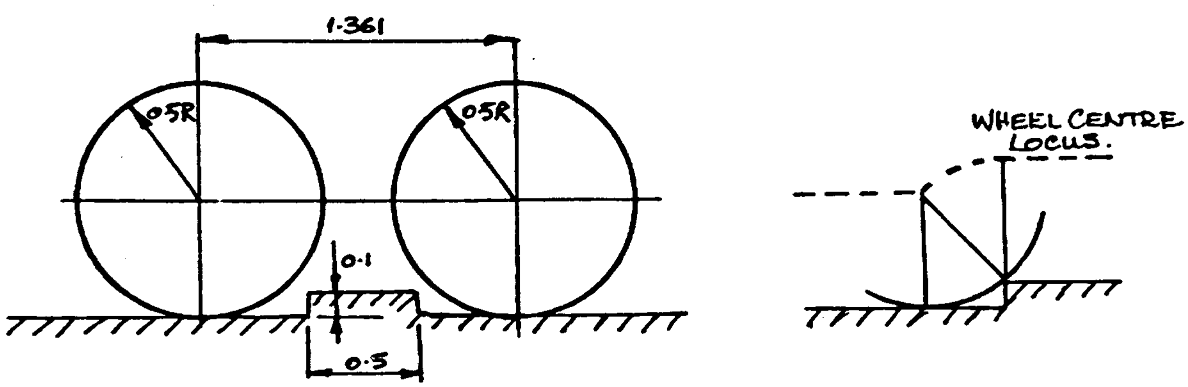

Figure 4 RAMP GEOMETRY
(dimension in m)

Figures 5 to 8 show typical response plots for various suspension parameters against time, for low speed case L01, in which the suspension takes about 15 seconds to pass over the road step.

Figure 5 represents the vertical motion at the centre of the suspension, (node 5 in Figure 3). The displacement is downwards relative to the unloaded state, and hence negative. The initial value at time zero is the static load response.

Figure 6 shows variations of the wheel load parameter, whilst 7 and 8 give the actual wheel loads. A perfect suspension would give a horizontal line for the latter two graphs, but as the calculations show, the trailer tilt which arises as each wheel successively lifts onto the step leads to unequal wheel loads. At this speed a steady state is achieved with each wheel on the step, with a rapidly damped oscillation at the start and end of the lift.

Figures 9 to 12 are the corresponding plots for a high speed run, where the step is traversed in about one third of a second. As would be expected much higher wheel loads are generated with greater inequality between the two.

General conclusions from Stage 3 were as follows:-

a) The dynamic analysis confirmed previous work in showing that good equalisation at low speed is obtained when the rotational stiffness of the equaliser bush is low.

b) Load equalisation at high speed is less effective than at low speed.

c) No untoward dynamic effects were excited with low bush stiffness.

Stage 4. Dynamic Non-linear Analysis

So far in the project it had been demonstrated that the use of equaliser bushes having low or zero values of stiffness and damping should give good equalisation, without introducing undesirable dynamic consequences. Unfortunately this did not appear to be the whole story, since there were reports of suspensions having equaliser links running on roller bearings, but which nevertheless suffered from poor equalisation. This leads to consideration of non-linear effects, such as friction and movement of spring contact points.

To investigate these matters a new dynamic response program was written which took account of non-linear geometric changes by updating the suspension geometry after each time step, and computing the movement of contact between spring and equaliser link. It also monitored the direction of slide at the contact points and brought into play opposing friction forces. The program has been applied to a typical suspension geometry and has revealed two further sources of poor equalisation.

Figure 13 shows a theoretical model of a twin axle, mechanically equalised, trailer suspension. As in previous models we consider symmetrical conditions only; that is, both sides of the trailer having identical motion. The model was chosen to represent one side only.

At the left hand end, the trailer is taken to be freely pivoting on the king pin. The trailer chassis frame is a rigid body connecting from the king pin to nodes 1,5,7 and 10. Joining nodes 2,5,6 is the equaliser link. This part can rotate about node 5, but is restrained by a torsional spring connecting the rotational freedoms of nodes 5 and 10. Elements 11,3,12 and 13,8,14 represent the road springs.

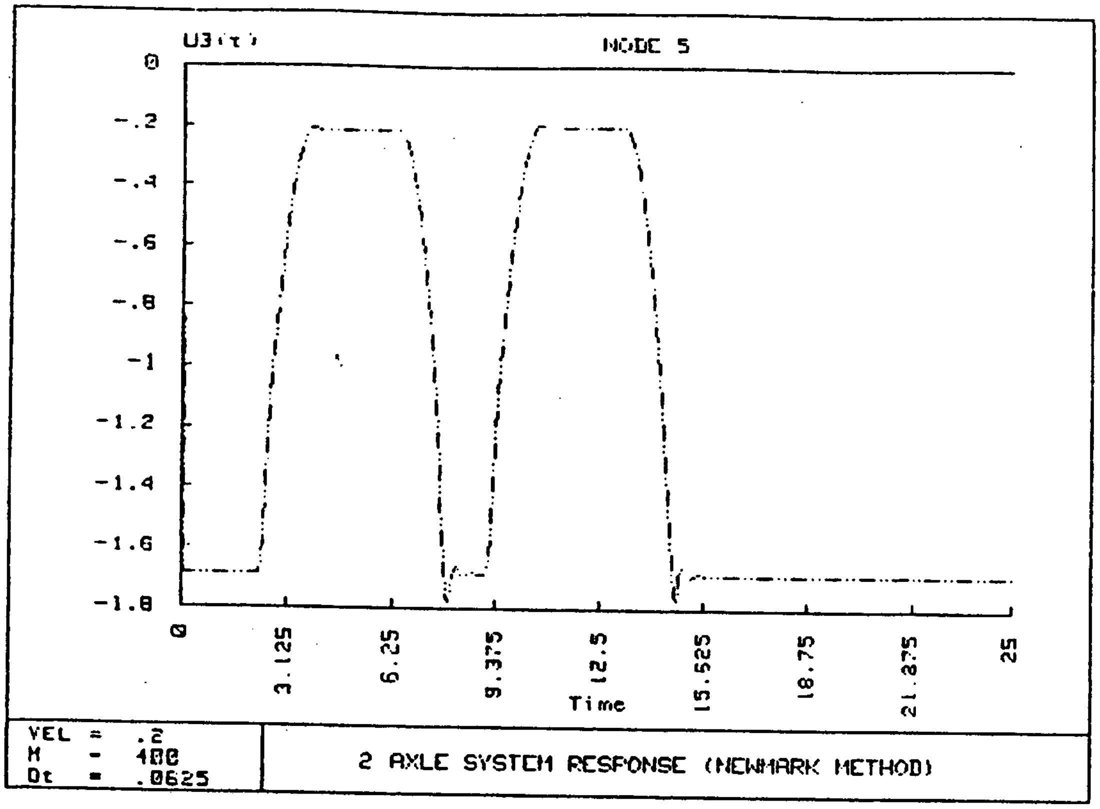

Figure 5

RUN L01

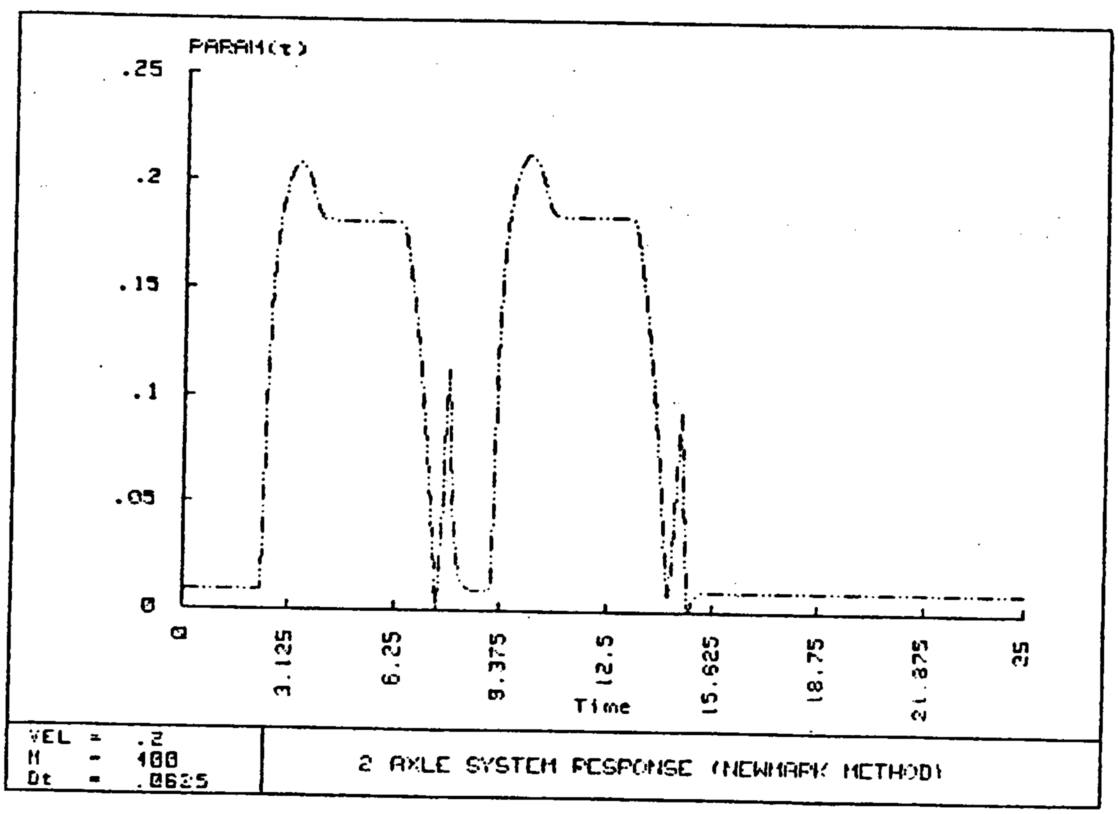

Figure 6

RUN L01

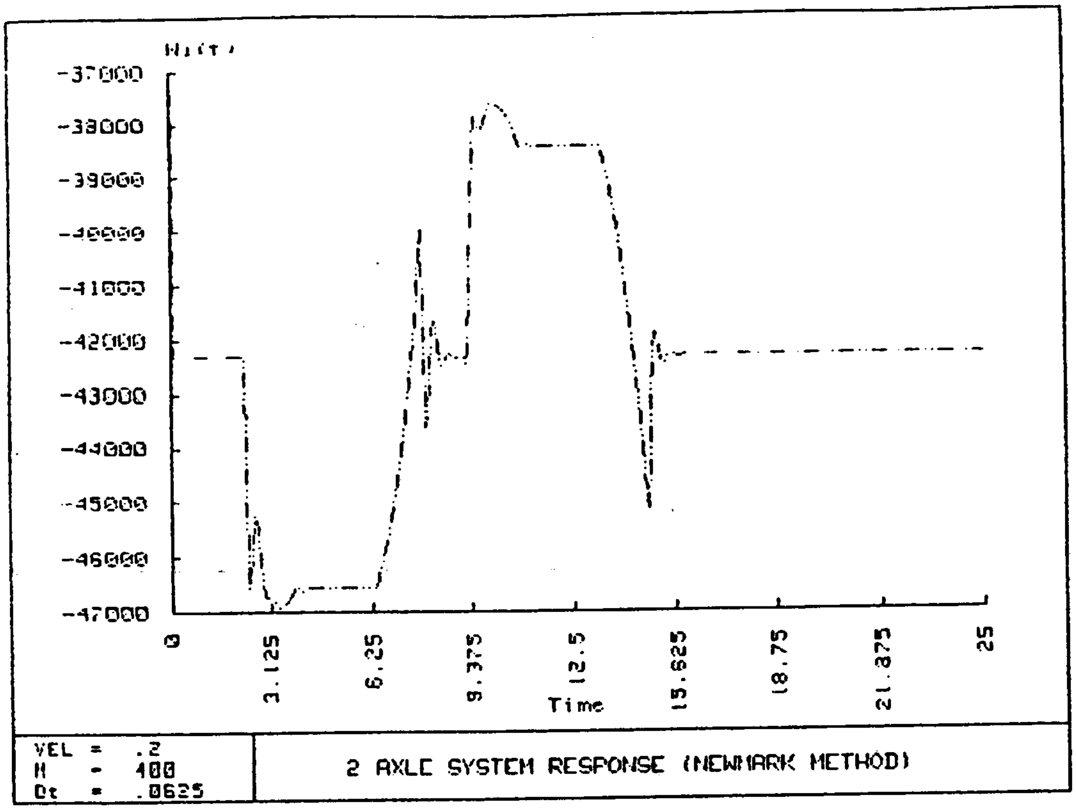

Figure 7

RUN L01

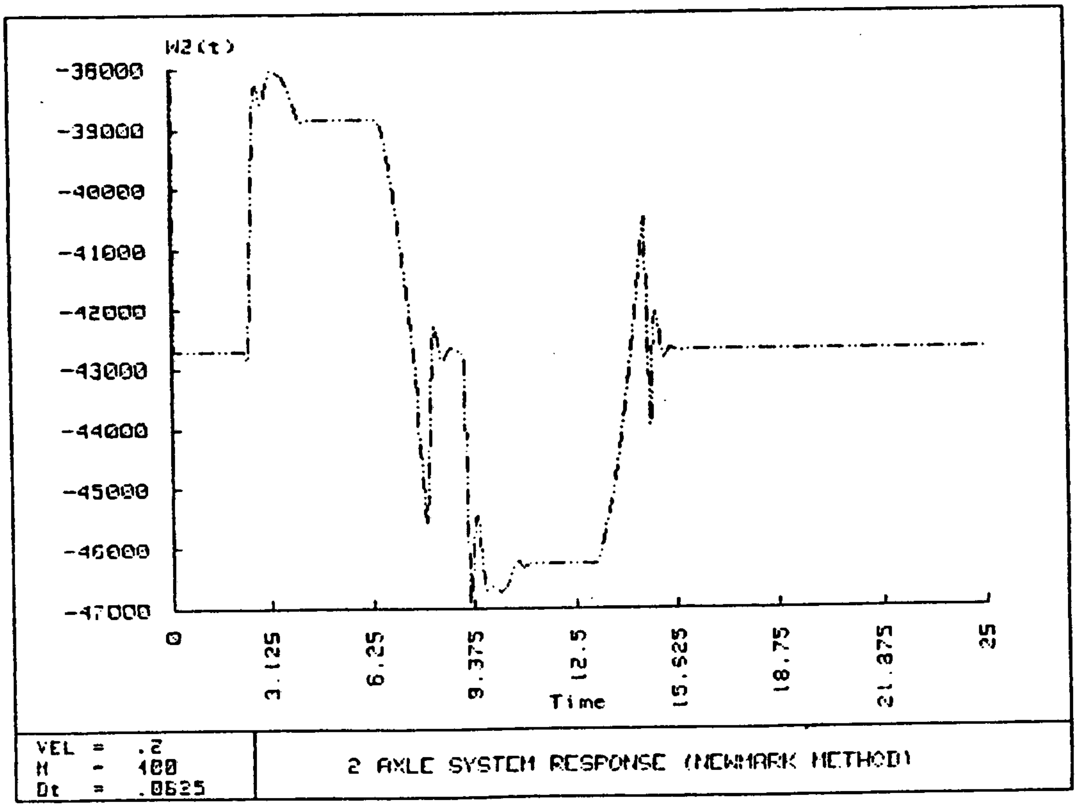

Figure 8

RUN L01

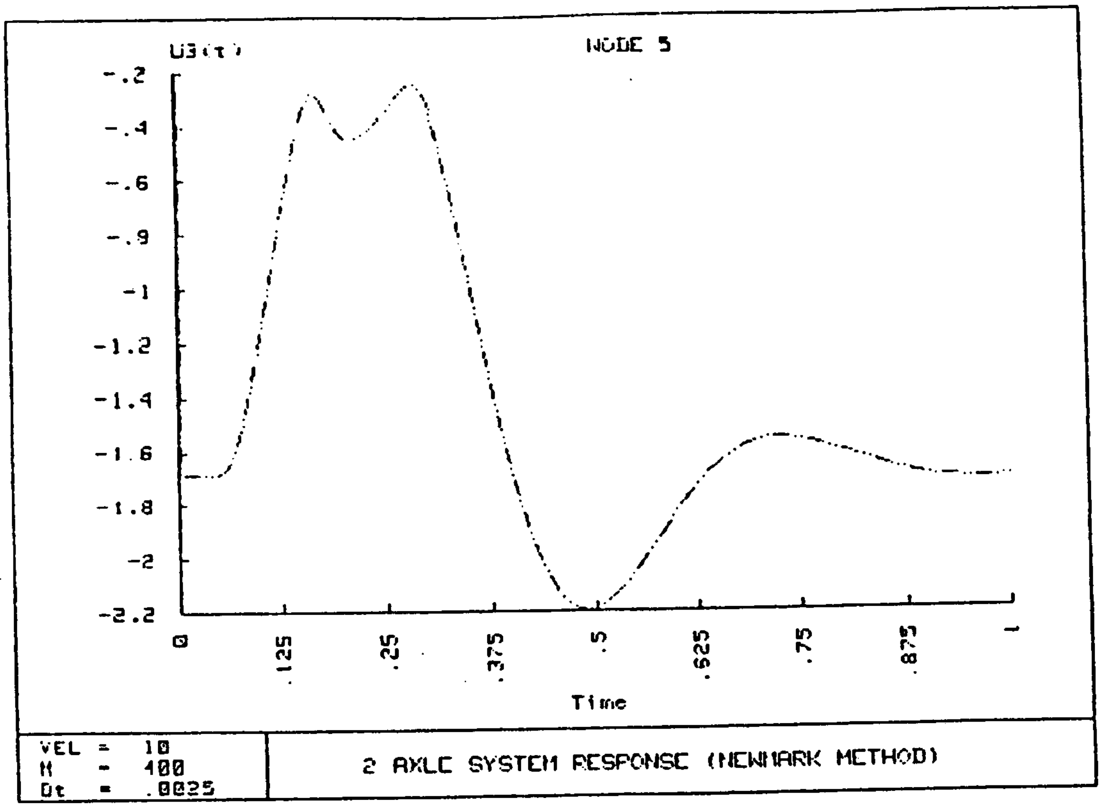

Figure 9

RUN L02

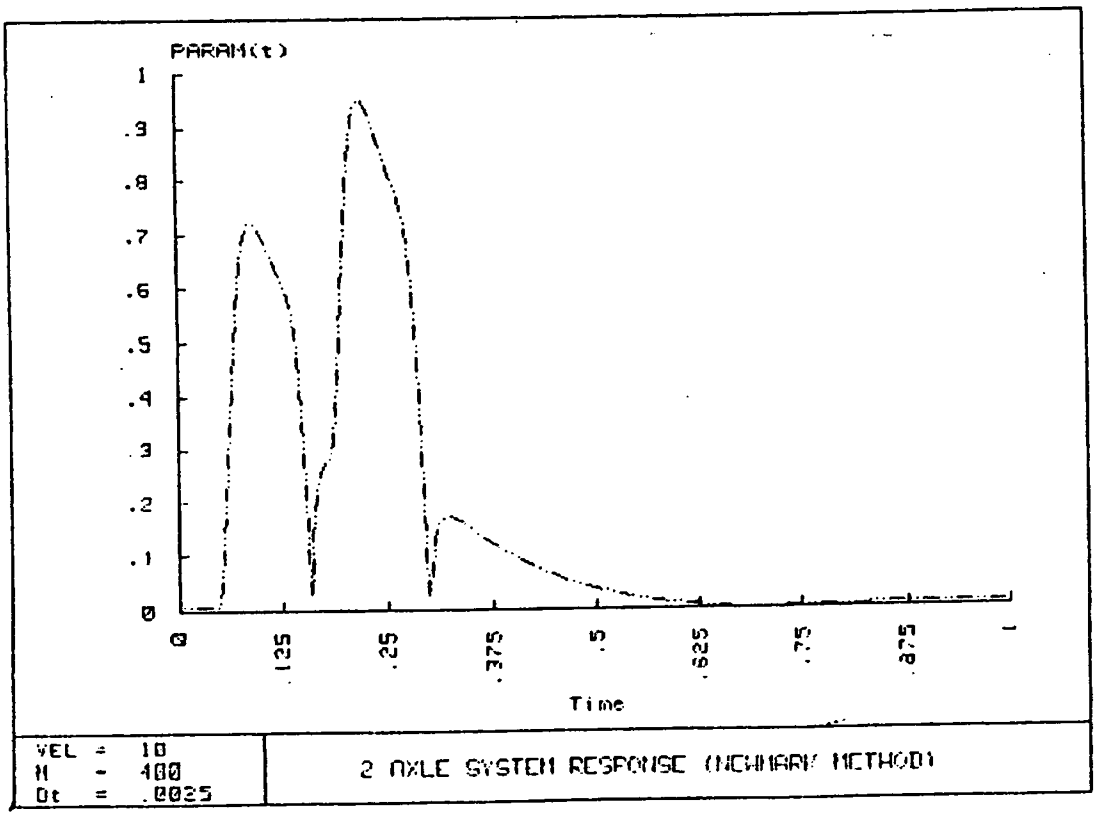

Figure 10

RUN L02

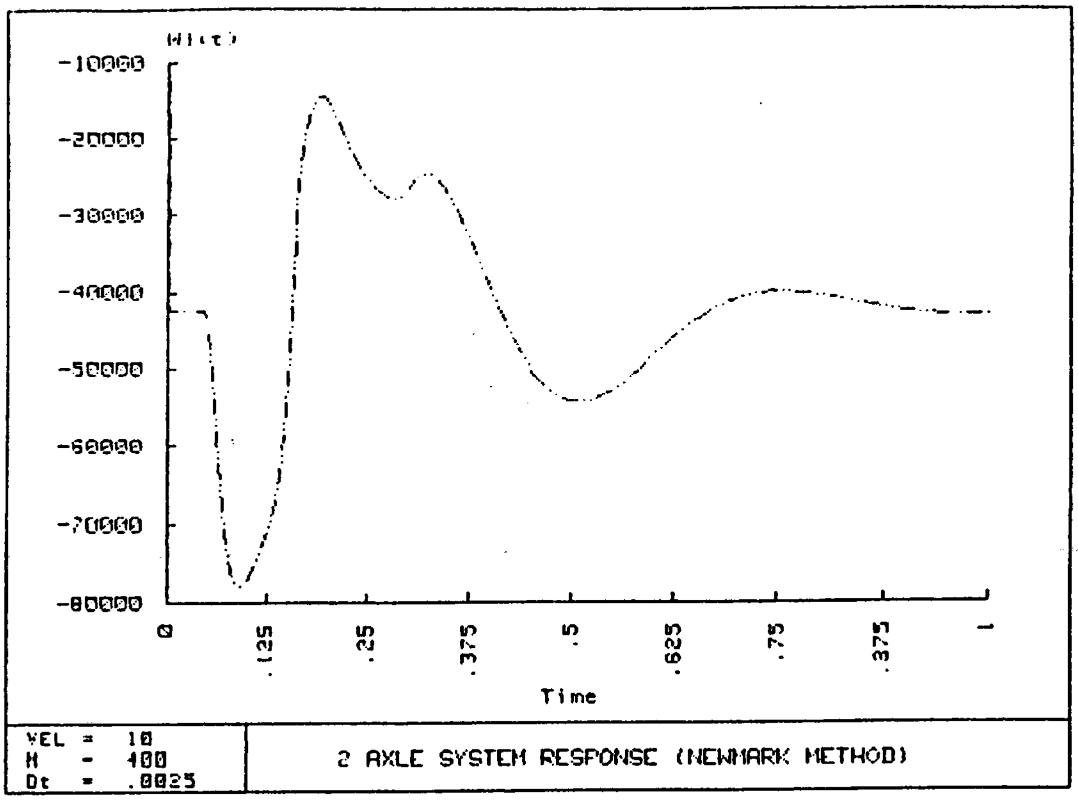

Figure 11

RUN L02

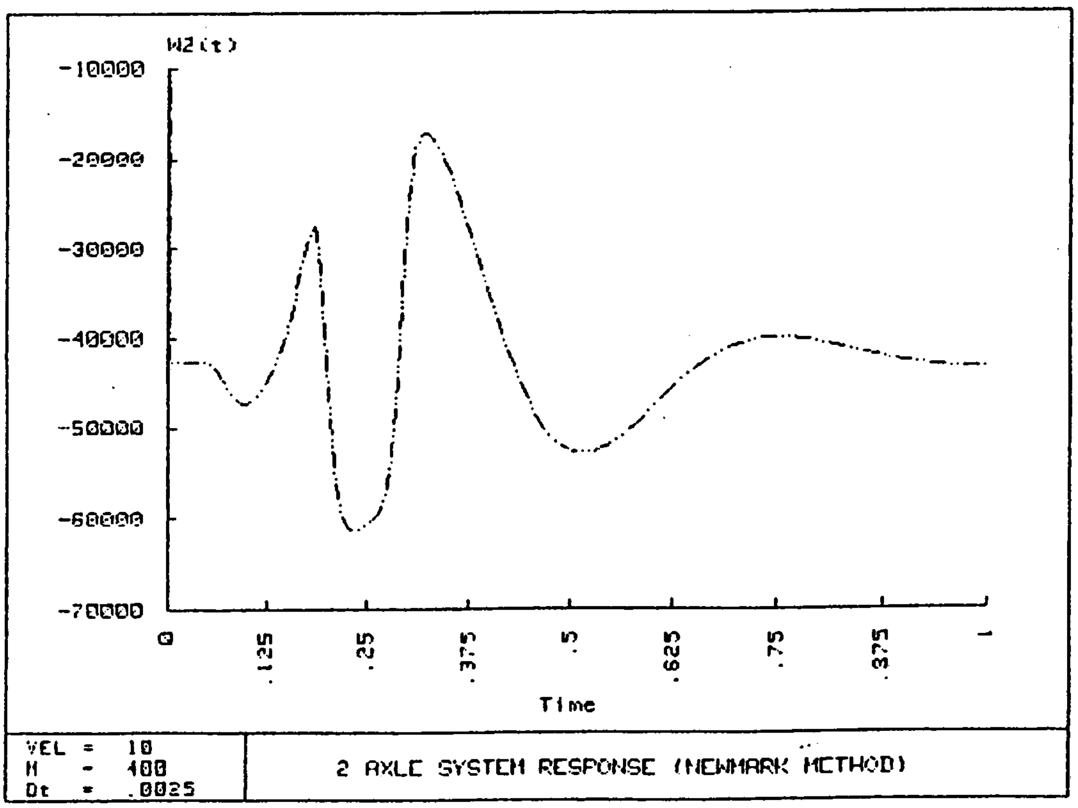

Figure 12

RUN L02

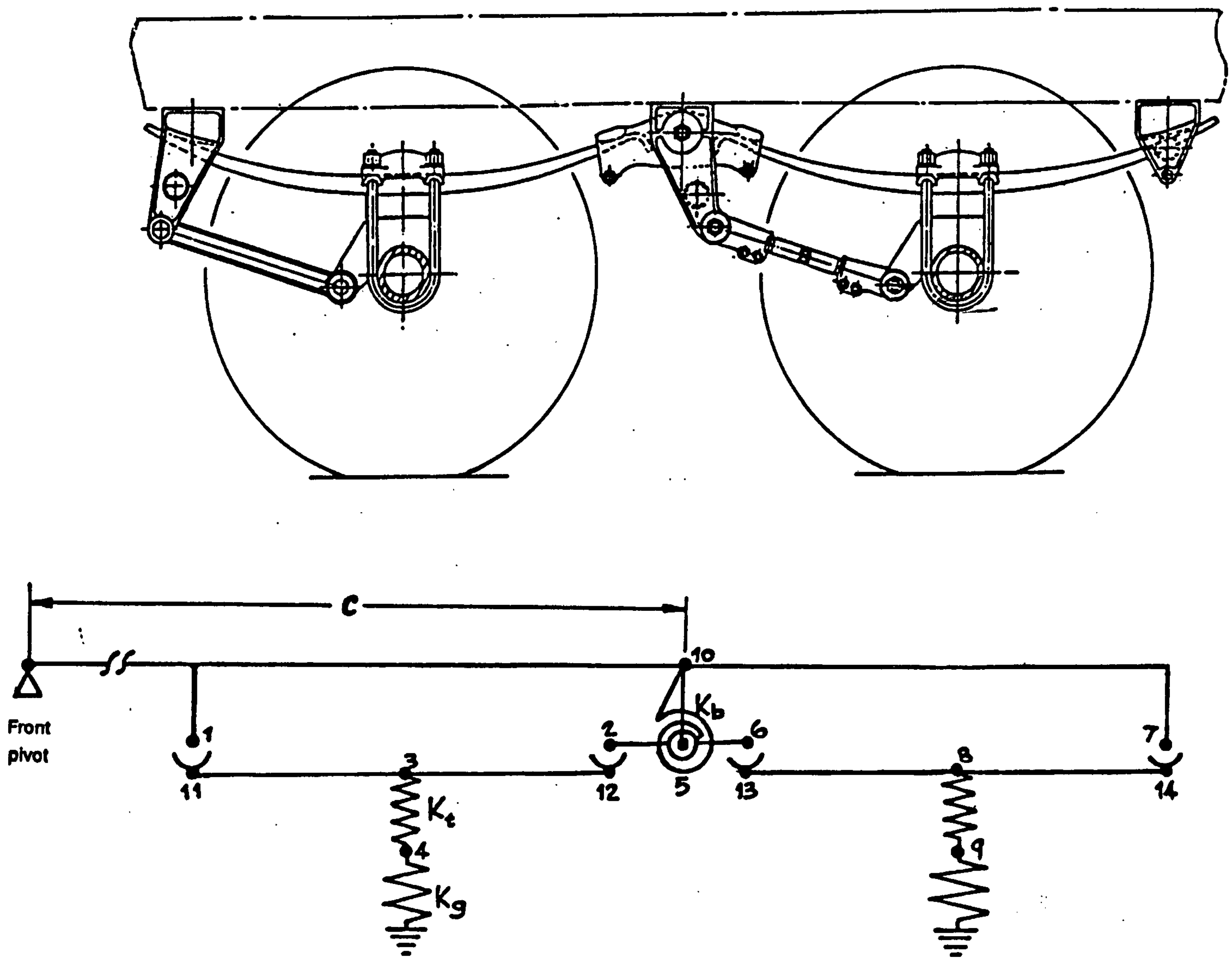

Figure 13 Typical equalised suspension
design and mathematical model.

Two further springs (3,4 and 8,9) allow tyre flexibility to be incorporated. Finally two very stiff springs connect nodes 4 and 9 to ground. The purpose of these latter springs is purely computational. They do not imply that the suspension behaviour is influenced by the dynamic properties of the road structure.

In addition to the usual elastic, inertia and damping forces which appear in the dynamic equations of motion, the model also takes account of friction forces at the contact points between springs and bearing pads. Furthermore, the change of contact position is fully allowed for as the motion takes the suspension through a history of different configurations.

Because of these extra effects, more degrees of freedom are required in the analysis than were previously necessary. The spring end displacements are now separated from the displacements of the contact pads so that they are capable of independent movement. Also, both horizontal and vertical displacements are now necessary in order to allow for the changing pattern of contact as the suspension changes configuration during motion.

The equations of motion retain the form of Equation 3, but now the matrices have 23 rows instead of 6.

The detail of the construction of the matrices is a matter of some complexity, but a full description may be found in reference 4. Here, however, it is sufficient to say that the non-linear contact between the springs and hangers is represented by constraint conditions derived from diagrams such as Figures 14a and 14b, in which the contact surfaces are idealised as cylindrical.

The solution procedure was based, as before, on the Newmark stepwise integration method, but modified to allow for the various non-linearities. However in the non-linear analysis the idea of making everything non-dimensional was dropped because of the additional complexity which this introduced.

The equations of motion represent equilibrium at the end of a linear timestep. Since the final contact position and direction of slip of the spring ends is unknown prior to solving a timestep, the calculation process has to be iterative within each timestep in order to achieve the correct balance of forces including friction. Also, the non-linearities due to changing geometry mean that the stiffness matrix has to be formed anew for each step. All this, together with the increase in matrix size, means that the non-linear solution is very much more time consuming than the linear problem.

Table 2 on ~~the following page,~~ presents a summary of the 29 analysis runs which have been carried out using the non-linear program. Blocks 1 and 2 were primarily for testing purposes, and to examine the stability and accuracy of the numerical processes. Block 3 consisted of 8 runs under the same step excitation conditions as the stage 3 work.

The most interesting comparison to make with this block of results is to take the non-linear case N10 and the linear solution L03. These are both low speed cases with low bush stiffness, and the results are illustrated by means of wheel parameter plots in Figures 15 and 16.

The linear result shows good equalisation over the two steady state regions which correspond to each wheel moving slowing over the step, although spikes occur in the plot at the times when a wheel rises over the front edge or falls off the rear edge. By contrast, the non-linear analysis shows a substantial difference in wheel loads, amounting to 2700N with the front wheel on the step, and 3200N when the rear wheel is lifted.

TABLE 2. LIST OF NON-LINEAR ANALYSIS RUNS

DESCRIPTION	JOB	SPRING END FRICTION	EQUALISER BUSH STIFFNESS	SPEED	RMS.	COMMENTS
1 Static load application and removal	N01	High	High	–		Wheel param at steady state P1 = 800
	N02	High	Zero	–		P1 = 720
	N03	Zero	High	–		P1 = 790
	N04	Zero	Zero	–		P1 = 680
2 Static load application & sinusoidal variation	N05	High	High	–		Wheel parameter at max. load P1 = 1060 min. load P1 = 380
	N06	Zero	Zero	–		max. load P1 = 1200 min. load P1 = 220
3 Simulation of short step in road profile (Payload 6500kg Ramp .025m)	N07	High	High	Low		Wheel parameter P1 = 2800/3500
	N08	High	Zero	Low		P1 = 2500/3100
	N09	Zero	High	Low		P1 = 3100/3600
	N10	Zero	Zero	Low		P1 = 2700/3200
	N11	High	High	High		P1 = 5700/7400 (peak)
	N12	High	Zero	High		P1 = 5700/7400 (peak)
	N13	Zero	High	High		P1 = 6000/7400 (peak)
	N14	Zero	Zero	High		P1 = 6000/7400 (peak)
4 True road profile	N15	High	High	Low	.00389	TRRL Track 1 Long equaliser arm
	N16	Zero	Zero	Low	.00321	
	N17	High	High	High	.00350	
	N18	Zero	Zero	High	.00342	
5 Additional runs (As item 3)	N19	Zero	Zero	Low		Linear constraints Wheel param 200/200
	N20	Zero	Zero	Low		Non-linear constraints Wheel param 1500/1800
	N21	Zero	Zero	Low		Long equaliser arm Wheel param 700/950
	N22	High	High	Low		Long equaliser arm Wheel param 700/900
	N23	Zero	Zero	Low		Contact point as TRRL Vehicle
	N24	High	High	Low		$\xi = .55$
	N25	Zero	Zero	Low		$\eta = .55$
	N26	High	High	Low		$\xi = .55$
	N27	Zero	Zero	Low		$\eta = -.55$
	N28	High	High	Low		Contact points as
	N29	=0.6	High	Low		TRRL Vehicle

Note High spring end friction was simulated using a coefficient of friction of 0.3.
High equaliser bush stiffness was simulated using a value of 1000 Nm/rad.

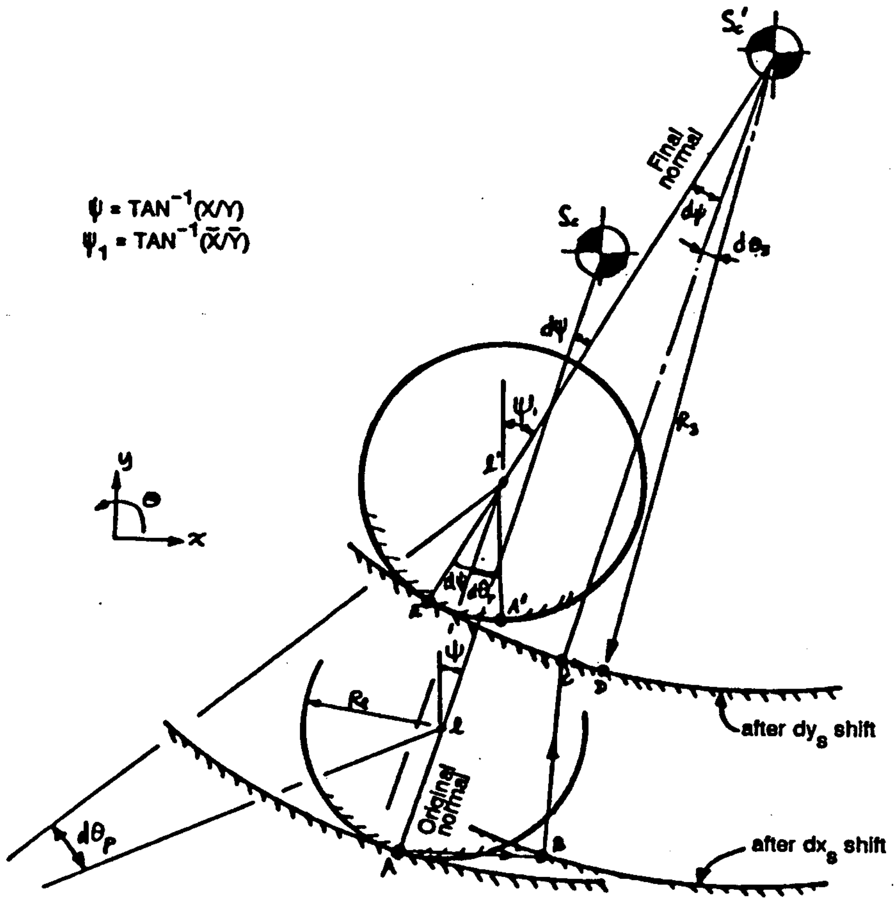

Figure 14a Geometry used for calculation of slip.

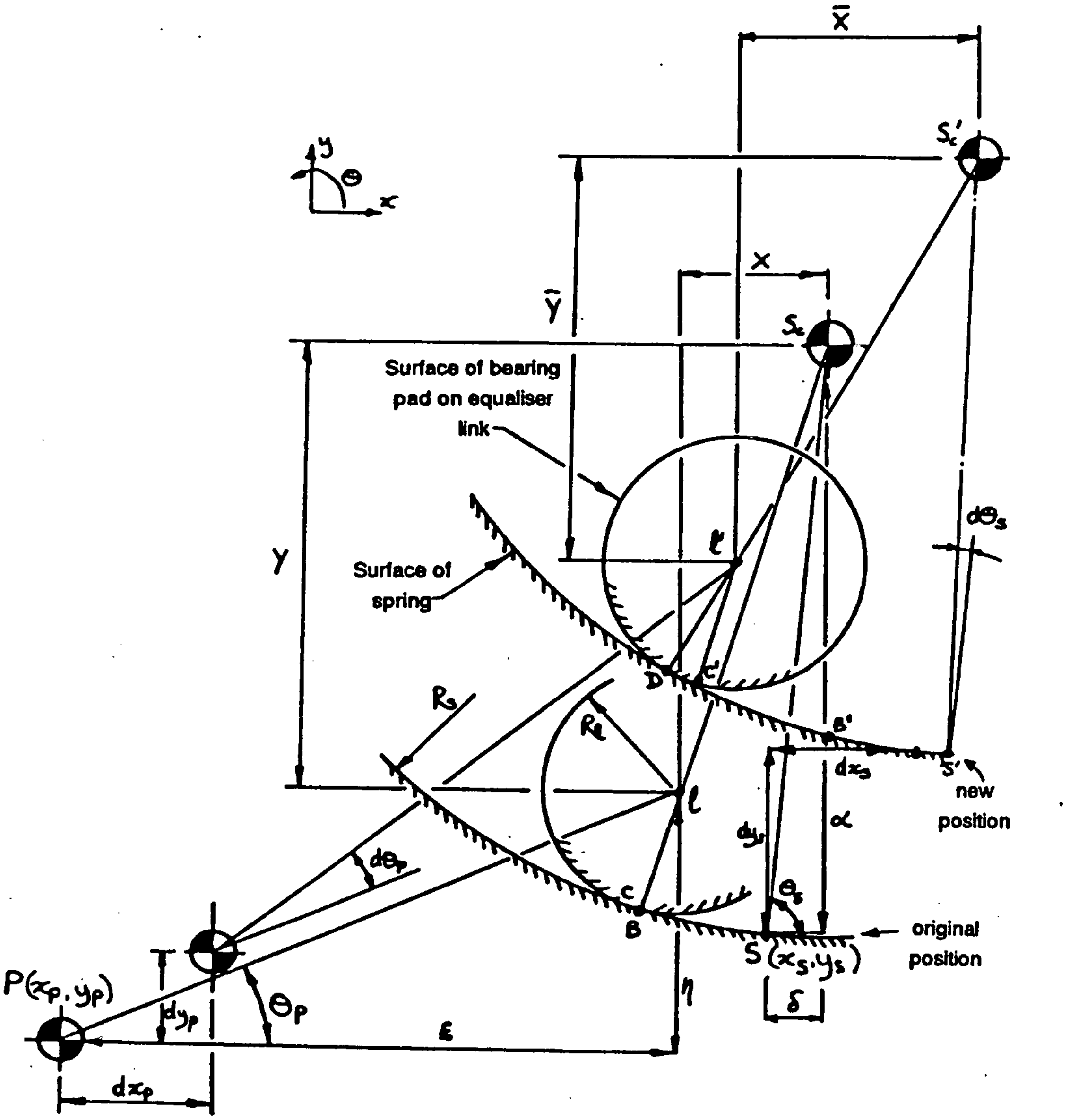

Figure 14b Geometry of contact surfaces

A careful dissection of the numerical values involved in the Newmark Integration revealed the cause of the lack of equalisation to be entirely the shift of contact point between spring end and equaliser link as these two components experienced relative rotation. This created unequal moment arms on the link, meaning that spring loads had to be unequal in order that equilibrium of moments could be achieved for the link.

In order to provide a second configuration of this diagnosis, an attempt was made to simulate linear behaviour within the non-linear program. This involved changes in the data and in the program. The modifications which were possible without major reconstruction of the software allowed a reasonably close approximation to linearity and gave the wheel parameter results shown in Figure 17. The fact that these look very similar to Figure 15 generates a substantial confidence in the correctness of the two programs and strongly confirms that movement of contact can be a very definite source of poor equalisation, ranking about equally with bush stiffness.

Block 4 of the non-linear results comprises 4 analyses of the response of the suspension to a 100m section of measured road profile. For these calculations the length of the equaliser arms was increased from previously used values in order to match more closely the geometry of a test trailer at TRRL, for which some test results were available. Incidentally an increase in link arm radius reduces the importance of contact point movement.

In order to give some overall measure of equalisation effectiveness over a time history of suspension response, a new parameter had to be defined. This is the item headed RMS in the table, and corresponds to

$$\text{RMS} = \left(\frac{1}{n} \sum_{1}^{n} \left(\frac{W_1 - W_2}{W_1 + W_2} \right)^2 \right)^{\frac{1}{2}} \qquad (5)$$

where W_1 and W_2 are instantaneous values of the two wheel loads at the end of a timestep, and n represents the number of timesteps.

The values in the table show that under low speed conditions equalisation is poorer with high friction and stiffness, by a factor of 21%. At high speed this factor is only a little more than 2%. This further supports the indication of previous calculation results, that poor equalisation is primarily a low speed and static problem. At high speeds, axle shock forces of large magnitude are generated by surface irregularities, and the lack of equalisation then has only a minor effect.

Figures 18 and 19 show two typical plots of road profile responses corresponding to low and high vehicle speed, respectively.

Block 5 contains a miscellaneous set of runs under step load conditions to investigate the effect of various changes in geometry and to determine whether friction forces acting tangentially at the spring contact surface could have a significant effect on equalisation. The main conclusion from these runs was that usual values of friction coefficient did not lead to a noticable effect, but on the other hand, an increase in friction coefficient to 0.6 did cause a deterioration in equalisation. Other runs showed also that changing the angles of the equaliser link arms did not make a very large difference to response.

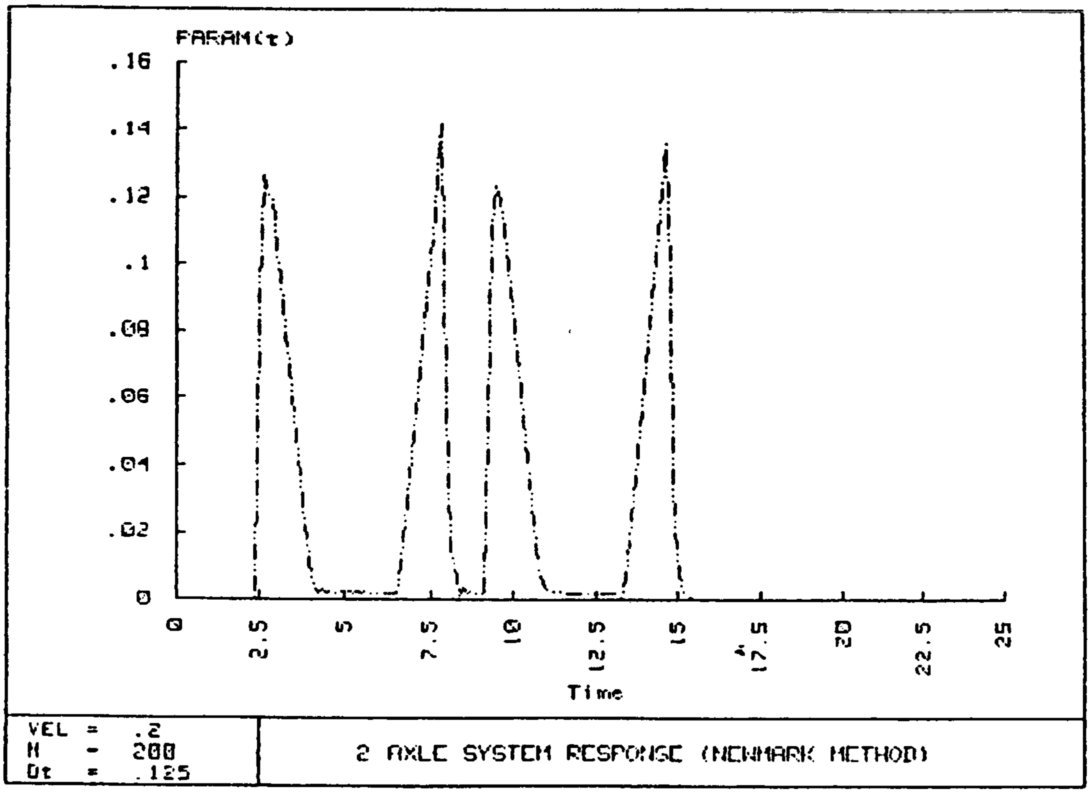

Figure 15

RUN L03

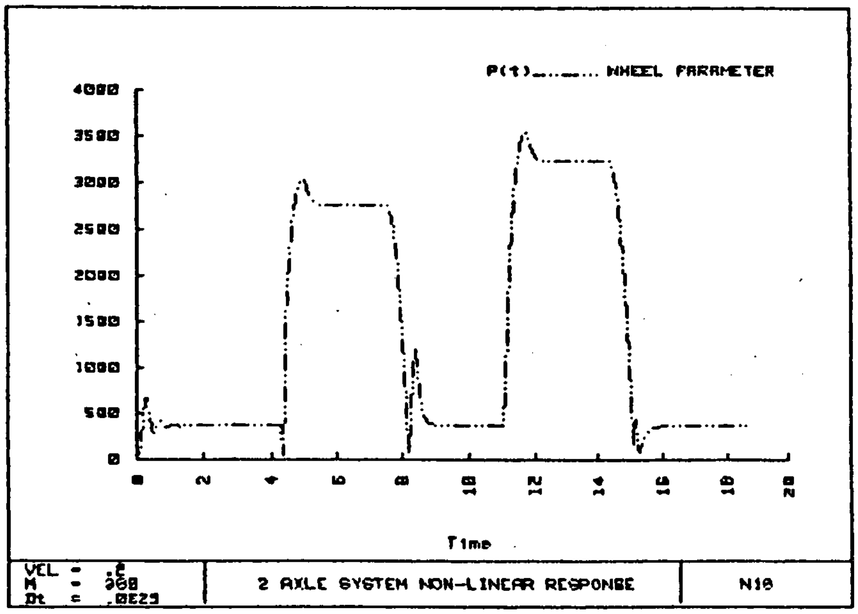

Figure 16

RUN N10

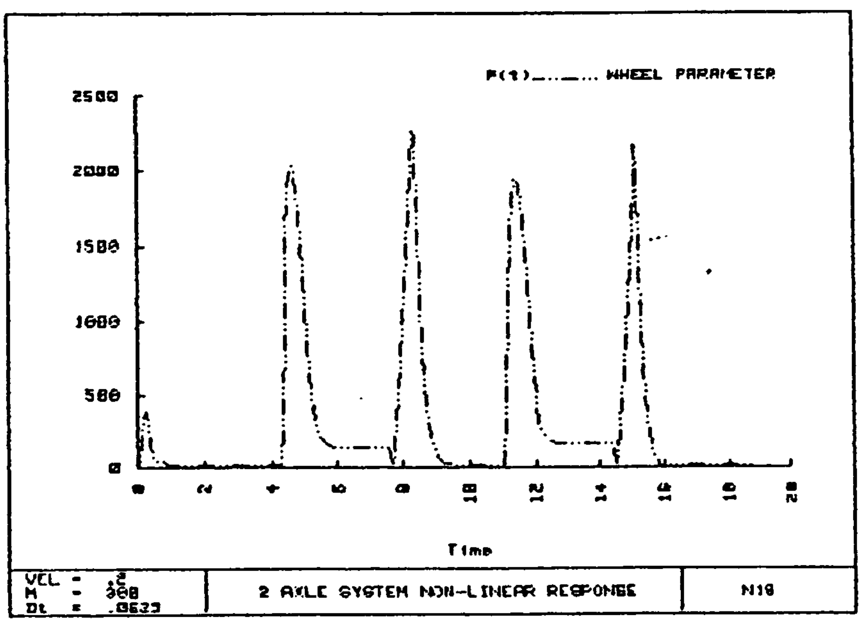

Figure 17

RUN N19

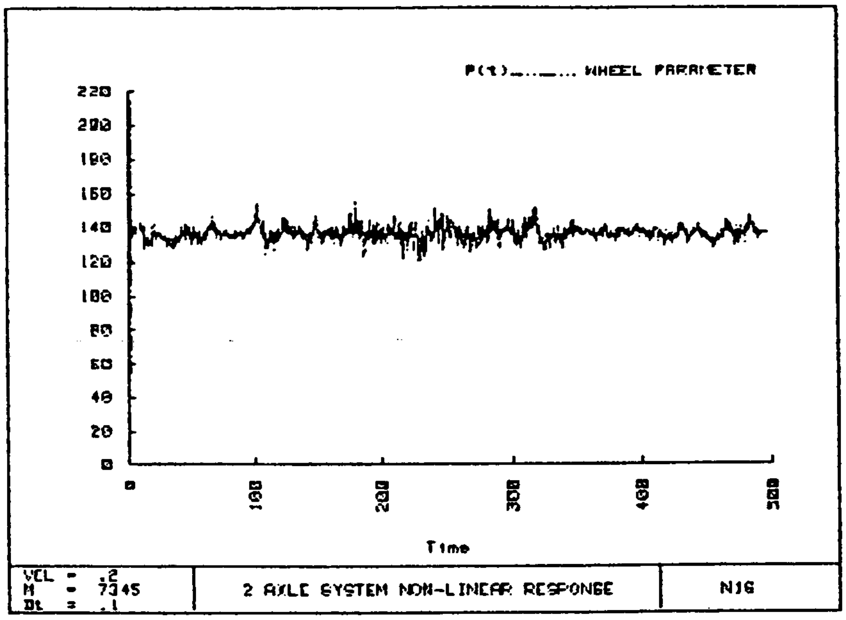

Figure 18

RUN N16

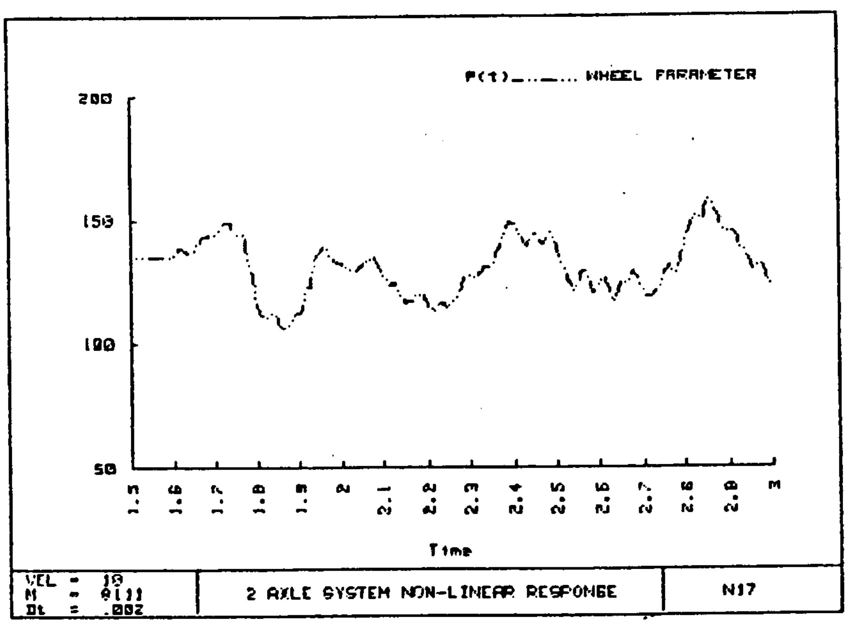

Figure 19

RUN N17

GENERAL CONCLUSIONS

1. Status of the Computer Program

The program works reliably and has satisfied numerous tests of accuracy and stability. It is now a practical tool for future further studies of the suspension problem.

However, during program development when making decisions on the means of representing friction forces, it was discovered that the application of beam theory to describe the behaviour of a leaf spring did not give good agreement with the limited available test evidence. Furthermore the correct way to theoretically treat the mixture of sliding and rolling contact at the spring ends was clouded by uncertainties. In order to reach a reasonable conclusion consistent with the project budget, it was decided to defer our attempt at a rigorous treatment of spring behaviour, and to incorporate a purely mathematical spring model. This appears to be a standard approach in vehicle dynamic analysis. In the present work this is not sufficient, however, because our theory needs to explicitly describe the effects of friction force external to the spring. An empirical description of friction force behaviour was adopted, which recognised changes in slip direction, and used the idea of a 'lagging' friction force to make the model behaviour coincide with simple calculable situations. The friction force assumption is believed to be an approximation of a similar level of accuracy to the spring load–deflection assumption.

These decisions allowed a working program to be produced, in which all the essential features of the suspension system were represented at a reasonable level of realism. On the other hand there is scope for more rigour. The achievement of this would require a careful investigation of spring behaviour in a combined experimental and theoretical development project.

2. Sources of Poor Equalisation

The study has revealed four possible reasons for poor equalisation, as follows:–

1. Rotational stiffness of rubber equaliser bushes.
2. Damping moment associated with rotation of equaliser bushes.
3. Change in moment arm caused by movement of contact points between the springs and their bearing pads.
4. Moment applied by friction forces to equaliser link.

All these effects, by introducing moments acting on the equaliser link, demand equilibrium states with unequal spring forces.

3. Test and Theoretical Evidence

Static experiments on the TRRL test trailer seem to indicate that reduction of friction at the spring ends considerably improved equalisation of wheel loads under sloping trailer conditions. To some extent the calculations are in conflict with this observation, because only with very high friction coefficient values, of the order of 0.6, does the theoretical model exhibit a significant dependence on friction. On the other hand, the detailed geometry of contact is likely to be important in respect of friction, and the correlation between the theoretical model and the actual geometry of the TRRL test trailer, is rather weak.

There may be another, unrecognised, contribution to the improved equalisation achieved during static test. Introducing a layer of low friction material between spring contact faces, may also provide a comparatively soft cushion which allows the contact pressure to be distributed over a larger area and may perhaps lead to pressure distributions whose centroid shifts less than is the case with metal-to-metal contact.

The limited theoretical dynamic analysis for road surface imperfections shows that equalisation is substantially improved at low speeds by removing the four inhibitors of equalisation. In contrast, at higher speed, they appear to make very little difference. The evidence of the dynamic vehicle measurements is inconclusive. The large differences between test and calculation may be due to differing physical properties or to other causes. In any case the resolution of differences between test and theory is likely to be resolved only by mounting a coherent program of tests and calculations, starting with simple conditions and culminating in full scale road tests.

4. Implications for Suspension Design

The theoretical model has demonstrated that to achieve good equalisation under static conditions the following design features should be aimed at:-

1. Freely rotating equaliser bearing.
2. Design of contact between spring and equaliser link must be such as to minimise the variation of moment arm of the spring end forces. Long equaliser links make this easier.
3. Friction between spring and equaliser contact surfaces must be minimised.

Of these the third appears to be the least significant.

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