ROAD RESEARCH LABORATORY

Ministry of Transport

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FORCES DUE TO TRAFFIC LOADS ON THE SHEAR CONNECTORS OF SIMPLY SUPPORTED COMPOSITE BRIDGES

by

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ABSTRACT

The loads due to traffic acting on the shear connectors of composite bridges (concrete slab on steel I-beam) were evaluated making use of Newmark's partial interaction theory. A number of examples were worked out for a 9.75 m (32 ft) span simply supported bridge loaded by the passage of a vehicle representative of the heaviest type permitted under the Construction and Use Regulations. The calculations were carried out for various degrees of connector stiffness found in practice.

As the amount of interaction is decreased, the loads acting on the shear connectors are reduced. The stiffest shear connection lead to reductions of 10%, 14% and 22% in shear ranges acting on connectors located at the end, quarter-span, and midspan sections respectively. The most flexible shear connection lead to corresponding reductions of 23%, 31% and 41%.

1. INTRODUCTION

In the past it was customary to build beam and slab bridges without any connection between the concrete slabs and the steel beams, and therefore as it was assumed that friction at the interface was negligible, no interaction occurred between the slab and the beams, the slab being free to slip over the beams. Nowadays composite bridges are built with a connection between the concrete and the steel in order to improve the efficiency of the structure. When the slab is bonded to the steel beam, complete interaction occurs and no slip takes place at the interface. When mechanical shear connectors (studs, channels, etc.) are used a certain amount of slip occurs when the face to face bond is broken at the interface, and only partial interaction takes place.

The degree of interaction depends on the stiffness of the connection. The stiffer the connection, the greater will be the shear force at the interface. In normal design calculations complete interaction is assumed in evaluating the forces on the connectors: in the present investigation the forces on the connectors are calculated using Newmark's partial interaction theory. The stiffness of shear connectors is established experimentally in push-out tests in which a length of joist is pushed relative to two concrete slabs shear-connected to it, and measurements of slip are taken at the interface (1). The modulus of the connector is then defined as the load per unit displacement.

The amount of slip which takes place at the interface of the slab and beam not only depends on the stiffness of the shear connector but also on the section properties of the slab and steel beam. The dimensionless parameter relating the stiffness of the shear connection to the stiffness of the beam and slab ($\frac{1}{C}$, see appendix) is therefore expressed in terms of the section properties, and the modulus and spacing of shear connectors. In the present investigation calculations have been done for values of 1/C equal to 20, 40, 100 and 200, where an increase in the value of 1/C denotes an increase in the stiffness of the connection. For composite Tee-beams the practical values of 1/C range from 20 to 100.

2. PARTIAL INTERACTION THEORY

The forces on the connectors may be calculated using Newmark's partial interaction theory (2) in which the following assumptions are made:

1. The shear connection is assumed to be continuous and uniform along the beam.
2. The load-deformation characteristic of the connectors is linear.
3. Plane sections remain plane in the slab and beam respectively.
4. No separation takes place at the interface.
5. The concrete is homogeneous and isotropic.

These assumptions only approximate to the actual conditions in a beam but tests have shown that for practical purposes the errors they introduce are generally fairly small (3). Since the assumption is made that the shear connection is continuous and uniform, the theory does not cater for beams in which bond at the interface has partly broken, along the length of the beam.

The theoretical solution consists of deriving a differential equation for the axial force in the slab and beam due to the action of the shear connectors. The sequence is

1. Slip is proportional to the shear at the interface, the spacing of the connectors and the modulus of the connectors.
2. The rate of change of slip along the length of the beam is equal to the strain difference at the interface.

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1. Slip is proportional to the shear at the interface, the spacing of the connectors and the modulus of the connectors.
2. The rate of change of slip along the length of the beam is equal to the strain difference at the interface.

3. The strains in the concrete and steel at the interface are expressed in terms of the axial force due to the shear connectors and the strains due to bending. Bending and axial force is obtained by combining 2 and 3.

4. The shear at the interface is equal to the rate of change of the axial force along the length of the beam.
By combining the results of 3 with 4 and 1 a differential equation is obtained for the axial force.

5. In order to satisfy equilibrium the sum of the internal moments is made equal to the external moment, and is substituted into the differential equation.

6. Since no uplift is assumed the curvatures of the slab and beam are equal and therefore a relationship exists between the bending in the slab and bending in the beam. This is also substituted into the differential equation.

The differential equation is now complete and can be solved for a particular external bending moment, which in this case has been taken as that produced by a point load anywhere along the span.

The boundary conditions are:

1. The axial force is zero at both ends of the beam and slab.
2. The shear force at the interface under the load point is continuous.

The solution of the differential equation gives values of the axial force (F) along the beam and therefore the interface shear may easily be calculated (c.f. appendix).

In the boundary conditions it is not assumed that the slip is zero at the load point, and in fact this only happens when the point load is on the midspan. Consequently when the load point is anywhere else along the span, the partial interaction shears on a short length, on the side of the load section nearer the midspan, are of opposite sign to those given by complete interaction as shown in fig. 2.

The effects of partial interaction are of a local nature and for a point load the distance at which the effects are negligible depend on the value of 1/C. Figure 3 shows the influence line for the force in the slab due to the end connector (assumed to be one hundredth of the span away from the end) for a value of 1/C equal to 100. The position of the load, at which the effect of partial interaction is negligible, is 0.16 of the span away from the section.

The influence lines of interface shears for different sections are shown in figures 4 to 11 for various values of 1/C. It may be seen that the partial interaction shear for the end section tends to zero as the load point moves towards the end support. For complete interaction it may be shown using simple theory that the interface shear tends to a maximum as the loan approaches the end support.

For sections near the support, negative values of q_c/q'_c, in figs. 4 to 11, tend to become large because q'_c tends to zero.

Under the action of a point load, simple theory suggests that for complete interaction two values of shear occur simultaneously at the load section, whereas partial interaction theory indicates that only one value occurs since there is no discontinuity in the slip distribution along the length of the beam. Further, if shear at any section is calculated as a load traverses that section, complete interaction shears show a sudden reversal in direction, whereas partial interaction shears show a gradual reversal.

3. FORCES DUE TO TRAFFIC LOADS ON THE SHEAR CONNECTORS

Design examples have been worked out for a 9.75 m (32 ft) span simply supported bridge. The shear connector loads were evaluated for the passage of a vehicle representative of the heaviest type falling within the Construction and Use Regulations.

This vehicle is composed of two front axles 1.22 m (4 ft) apart and two back axles also 1.22 m (4 ft) apart. The overall distance between the front and back axles is 4.57 m (15 ft). The front axles carry a load of 40 kN (4T.), and the back axles each carry a load of 90 kN (9T.) In the calculations it was assumed that due to the transverse spacing of the composite beams a solitary beam carried half the total weight of the Class A vehicle.

The shear at the end section of a beam is shown by simple theory to be a maximum for complete interaction when the outer heavy wheel load is just on the end of the span. For partial interaction the maximum occurs when the heaviest wheel load is between 0.06 and 0.14 of the span away from the section considered (fig. 12).

The shear at the quarter span and midspan sections show a similar behaviour to the shear at the end section when partial interaction is considered (fig. 13 and 14). In all cases the increase and decrease in shear is gradual as the vehicle moves across a section.

The results from the design examples are shown in table 1. The greatest reduction in range of shear due to partial interaction is shown to be at midspan.

Calculations for other types of vehicles and bridges may be done using the influence lines given in fig. 4 to 11.

4. CONCLUSIONS

If the forces on shear connectors are calculated for complete and partial interaction, the following trends may be observed:

1. Reductions in the forces acting on the shear connectors of a composite beam due to partial interaction depend on the location of the point of application of the load with respect to the connector under consideration. When the load is sufficiently far away from the section, no reductions take place.
2. The distance at which no reduction takes place depends on the stiffness of the shear connection and the section properties of the composite beam.
3. The magnitudes of the reductions also depend on the stiffness of the shear connection and the section properties.
4. The shear at the end sections of composite beams tends to zero as a result of incomplete interaction when a point load approaches the end sections.
5. Sudden reversals of shear do not take place at any section under the action of a traversing load when partial interaction is considered.
6. The example with the stiffest shear connection met in practice shows reductions of 10%, 14% and 22% in shear ranges of connectors located at the end, quarter span, and midspan sections respectively.
7. The example with the most flexible connection met in practice shows reductions of 23%, 31% and 43% of the end, quarter span and midspan sections respectively.

5. ACKNOWLEDGEMENTS

The work described in this report is part of a programme of research of the R.R.L., which was carried out under the supervision of Dr. J.C. Chapman in the Engineering Structures Laboratories of Imperial College.

6. REFERENCES

1. J.S. TERASZKIEWICZ - "The effects of partial interaction on the longitudinal stresses in composite T-beams". Road Research Report No. 174, 1968.

2. C.P. SIESS, I.M. VIEST, M.N. NEWMARK - "Studies of slab and beam highway bridges - part 3" Univ. of Illinois Eng. Exp-Stn Bul. 396, 1952.

3. J.C. CHAPMAN and S. BALAKRISHNAN - "Experiments on composite beams" J. Inst Str. Eng. Nov. 1964 Vol. 42 No. 11.

NOTATION

Subscripts

s = slab

b = beam

Notation

A_s, A_b = cross-sectional areas of slab and beam

$\frac{1}{C}$ = dimensionless parameter relating the stiffness of the shear connection to the stiffness of the beam and slab (see Appendix)

E_s, E_b = modulus of elasticity of slab and beam

I_s, I_b = moment of inertia of slab and beam

k = modulus of shear connector

L = span length of beam

q_c = horizontal shear per unit length of the beam at the steel-concrete interface for partial interaction.

q'_c = horizontal shear per unit length of the beam at the steel-concrete interface for complete interaction

s = spacing of shear connectors

u = distance from the support of a concentrated load

x = distance from the support of a cross-section

z = distance between the neutral axes of the slab and beam

V = vertical shear force

F = horizontal axial force acting at the neutral axis of the slab and beam.

M = external moment applied to the composite beam.

APPENDIX

PARTIAL INTERACTION THEORY

The following is an extract from Newmark's theory$^{(2)}$. A solution for the partial interaction shear at the interface is found as a ratio of the complete interaction shear (q_c/q'_c). Influence lines of this ratio are shown in figs. 4 to 11 for $\frac{1}{C}$. = 100, 200, 20 and 40.

$$\frac{d^2F}{dx^2} - F\,\frac{k}{s}\,\frac{\overline{EI}}{\overline{EA}\Sigma EI} = -\,\frac{k}{s}\,\frac{Mz}{\Sigma EI}$$

for $x \leqslant u$

$$\frac{q_c}{q'_c} = 1 - \frac{\sinh\frac{\pi}{\sqrt{C}}\left(1 - \frac{u}{L}\right)}{\left(1 - \frac{u}{L}\right)\sinh\frac{\pi}{\sqrt{C}}}\cosh\left(\frac{\pi}{\sqrt{C}}\,\frac{x}{L}\right)$$

The following expressions have been introduced for convenience:-

$$\frac{1}{\overline{EA}} = \frac{1}{E_sA_s} + \frac{1}{E_bA_b}$$

$$\Sigma EI = E_sI_s + E_bI_b$$

$$\overline{EI} = \Sigma\,EI + \overline{EAz^2}$$

$$C = \frac{s}{k}\,\frac{\pi^2\,\overline{EA}\Sigma EI}{L^2\,\overline{EI}}$$

and for $x \geqslant u$

$$\frac{q_c}{q'_c} = 1 - \frac{L\sinh\frac{\pi}{\sqrt{C}}\,\frac{u}{L}}{u\sinh\frac{\pi}{\sqrt{C}}}\cosh\left[\frac{\pi}{\sqrt{C}}\left(1 - \frac{x}{L}\right)\right]$$

The interface shear for complete interaction is:

$$q'_c = V.\,\frac{\overline{EA}\,z}{EI}$$

the interface shear for incomplete interaction is:-

$$q_c = \left(\frac{q_c}{q'_c}\right).\quad q'_c = \left(\frac{q_c}{q'_c}\right)V.\,\frac{\overline{EA}\,z}{\overline{EI}}$$

TABLE 1

Ratios of partial and Complete interaction interface shears in simply supported beam under vehicle loading

Section	1/C = 200		1/C = 100		1/C = 40		1/C = 20		Max % reduction in range
	shear ratio (q_c/q'_c)		shear ratio (q_c/q'_c)		shear ratio (q_c/q'_c)		shear ratio (q_c/q'_c)		
	(+)	(-)	(+)	(-)	(+)	(-)	(+)	(-)	
End	1.0	0.90	1.0	0.87	1.0	0.82	1.0	0.76	23.6
Quarter-span	0.59	0.89	0.48	0.86	0.28	0.80	0.11	0.75	31.0
Midspan	0.80	0.73	0.76	0.69	0.66	0.61	0.58	0.56	42.8

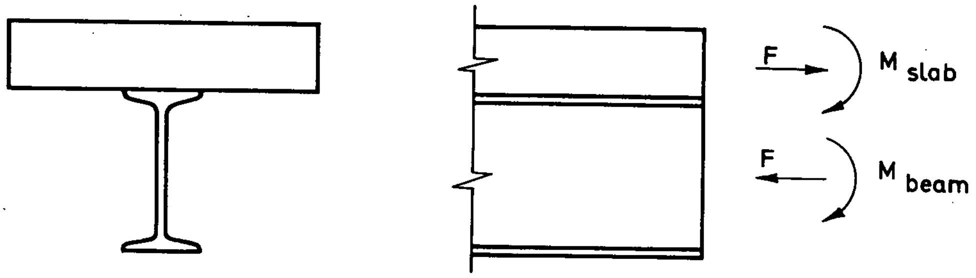
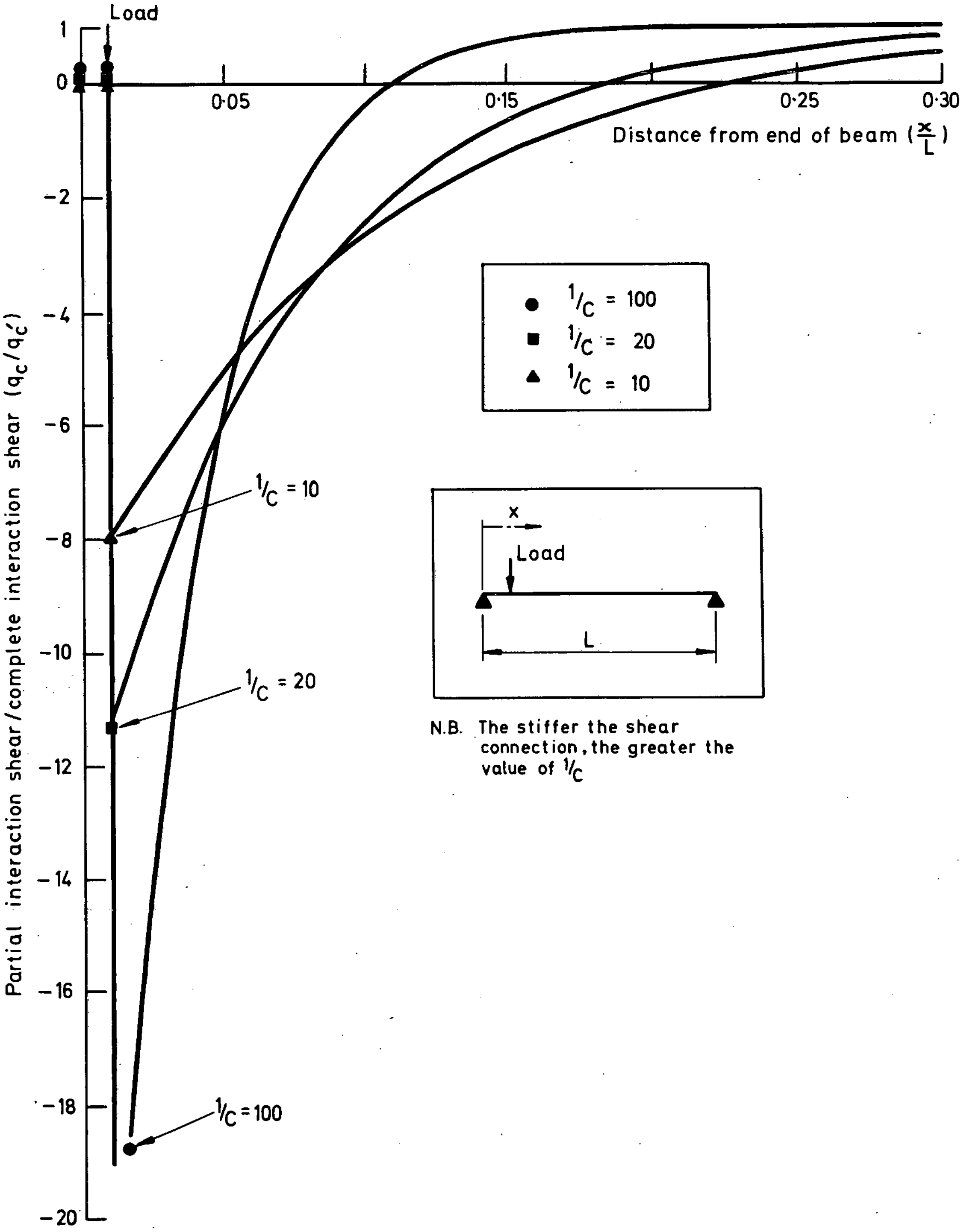


Fig.1. INTERNAL FORCES AND MOMENTS IN COMPOSITE T-BEAM.



N.B. The stiffer the shear connection, the greater the value of $1/C$

Fig. 2. INTERFACE SHEAR IN SIMPLY SUPPORTED BEAMS WITH PARTIAL INTERACTION (POINT LOAD AT x/L = 0·01)

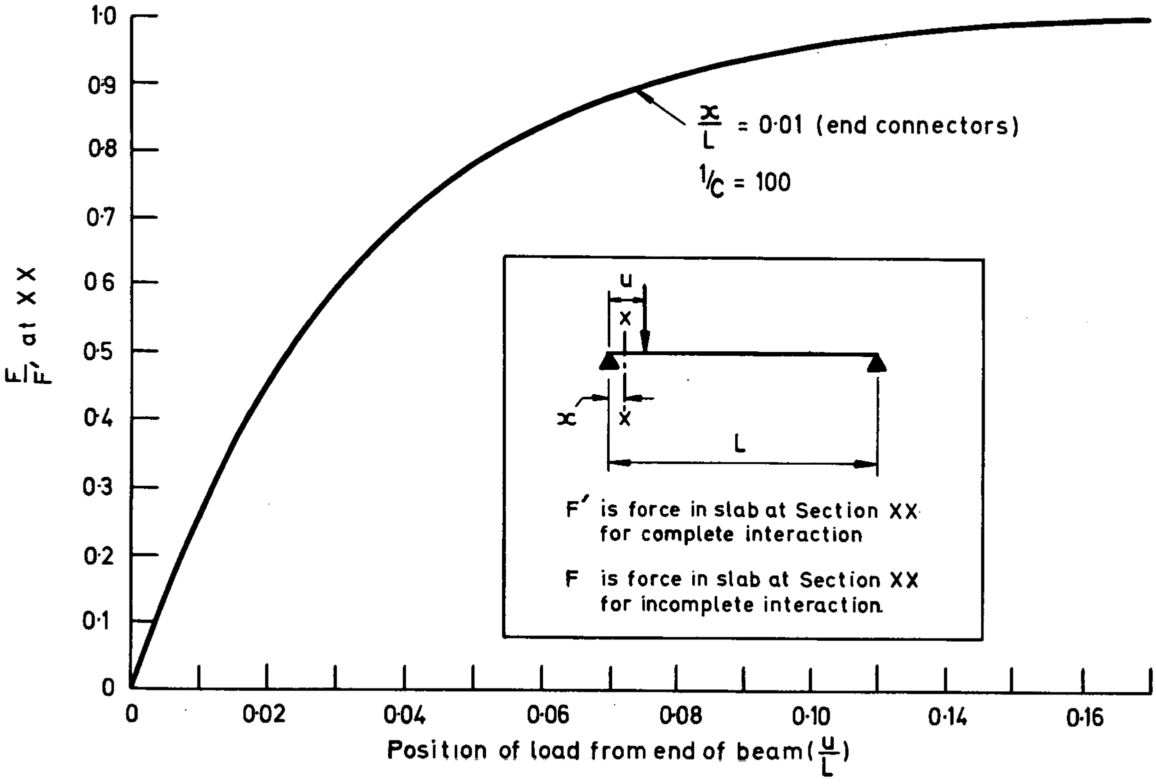


Fig. 3. INFLUENCE LINE FOR FORCE IN SLAB FOR PARTIAL INTERACTION AT END CONNECTORS

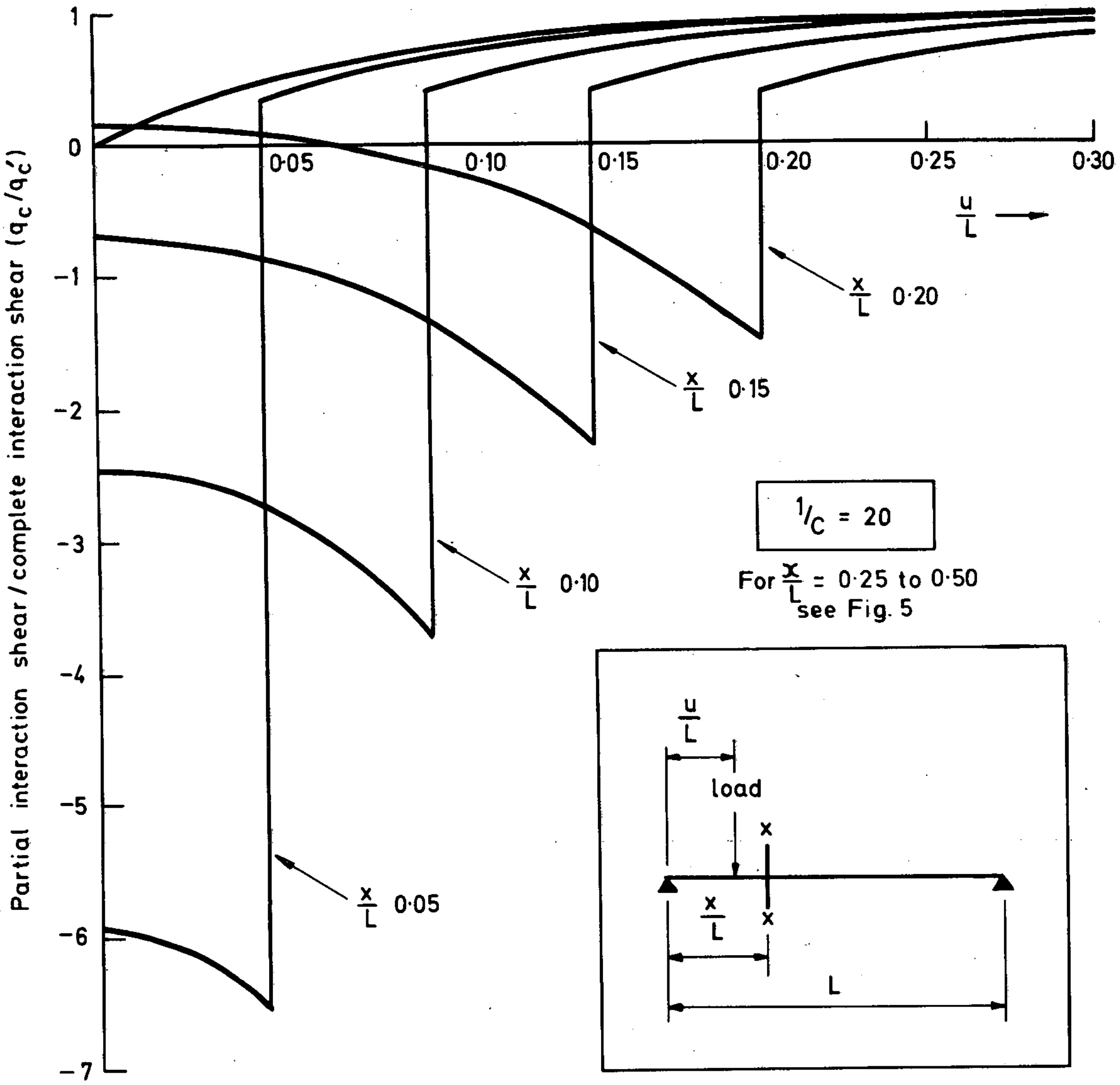


Fig. 4. INFLUENCE LINES FOR INTERFACE SHEAR WITH PARTIAL INTERACTION ($1/C$= 20)

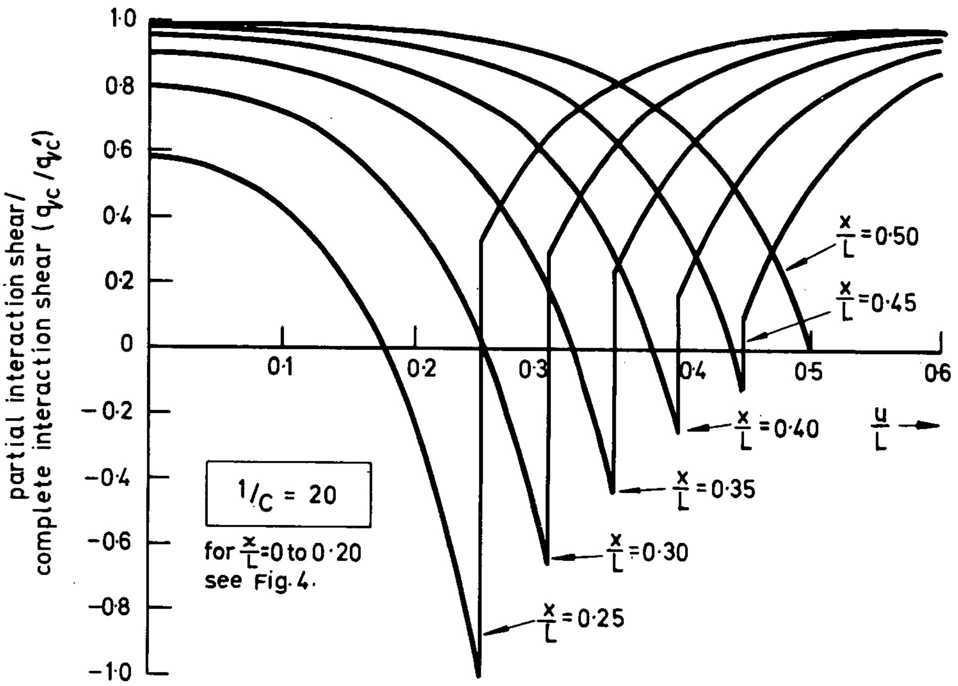


Fig. 5. INFLUENCE LINES FOR INTERFACE SHEAR WITH PARTIAL INTERACTION ($1/c$=20)

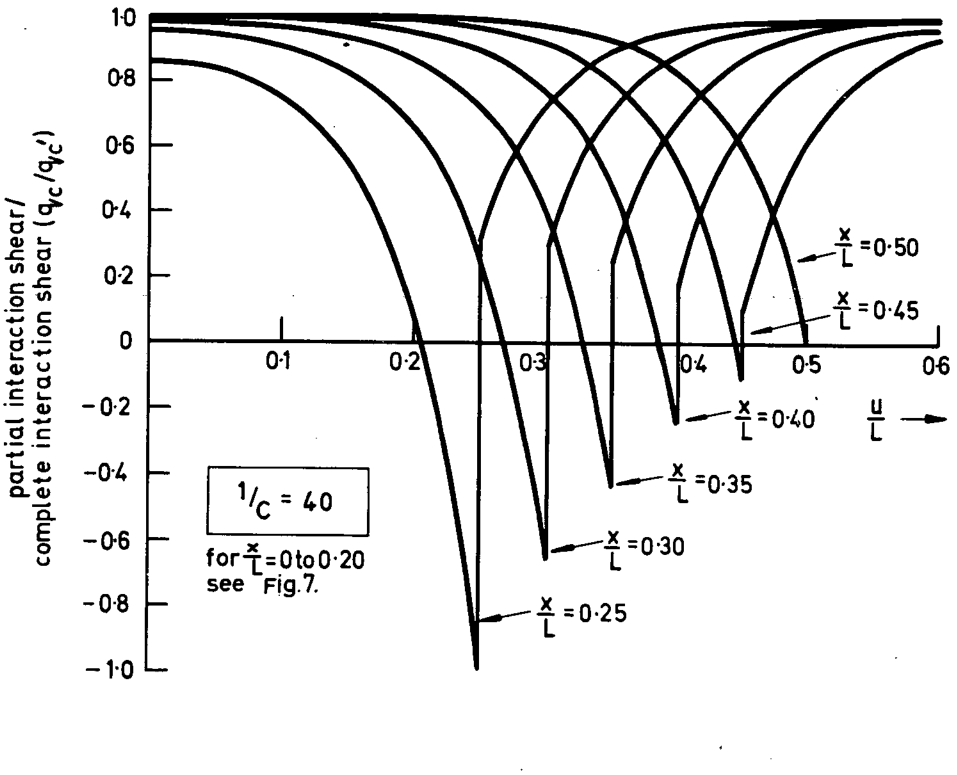


Fig. 6. INFLUENCE LINES FOR INTERFACE SHEAR WITH PARTIAL INTERACTION ($1/c$=40)

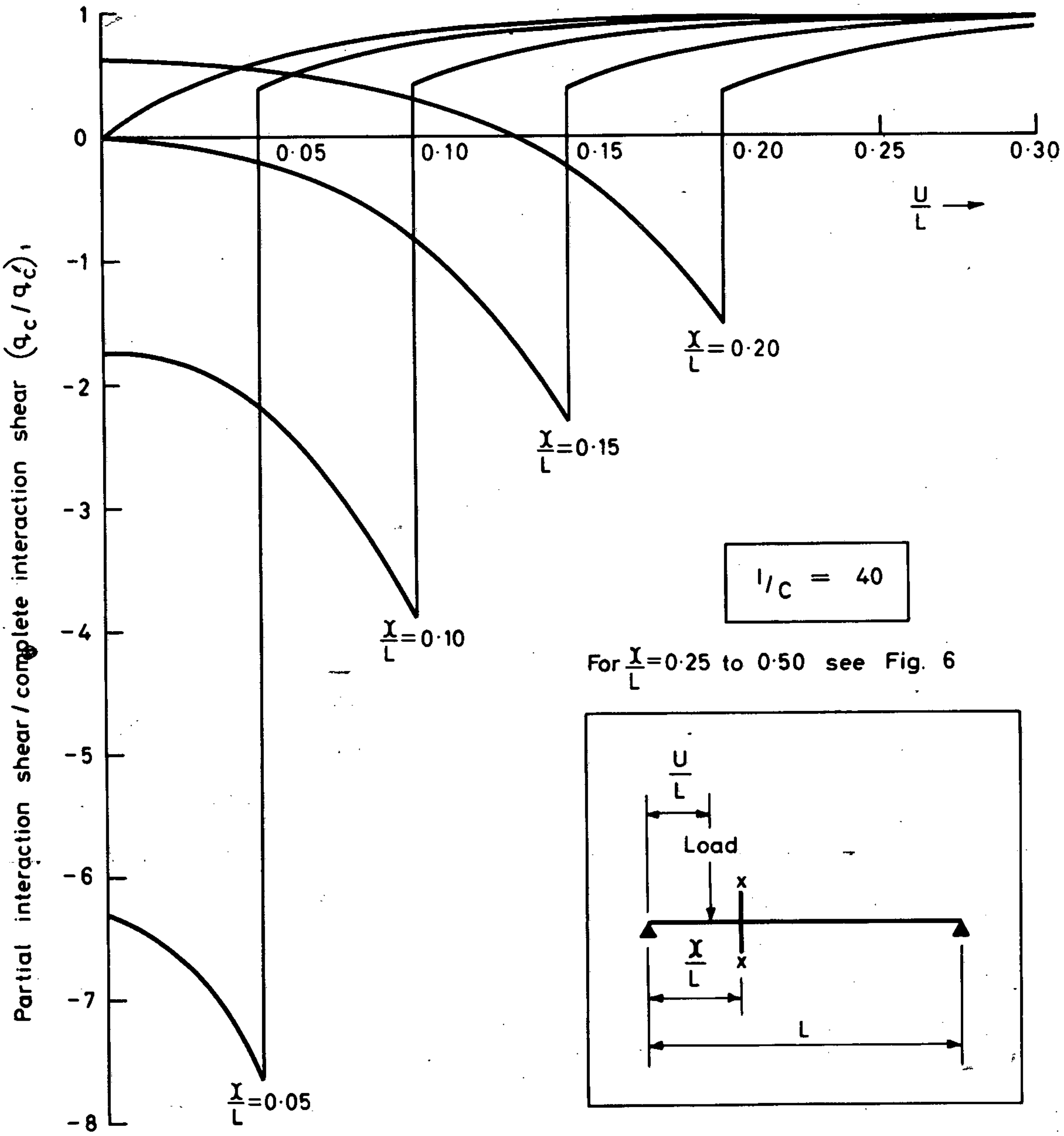


Fig. 7. INFLUENCE LINES FOR INTERFACE SHEAR WITH PARTIAL INTERACTION $(l/C = 40)$

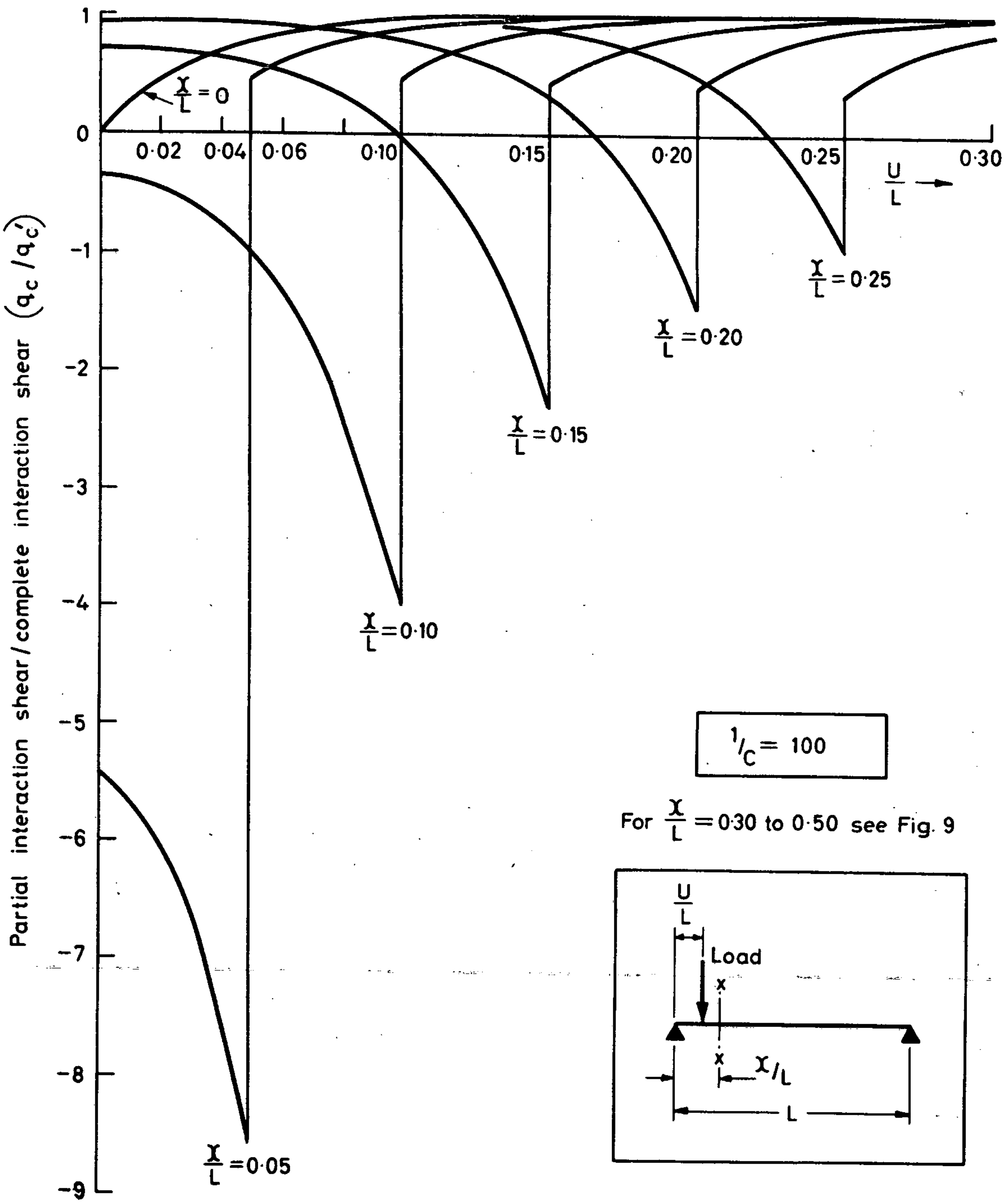


Fig. 8. INFLUENCE LINES FOR INTERFACE SHEAR WITH PARTIAL INTERACTION $(1/C = 100)$

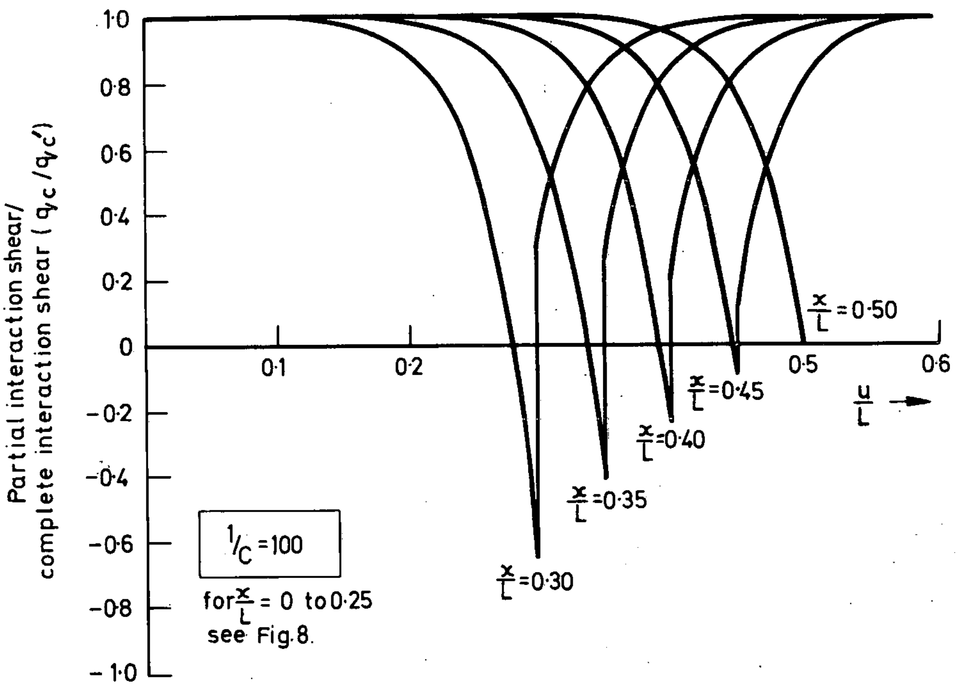


Fig. 9. INFLUENCE LINES FOR INTERFACE SHEAR WITH PARTIAL INTERACTION ($^1/_C$=100)

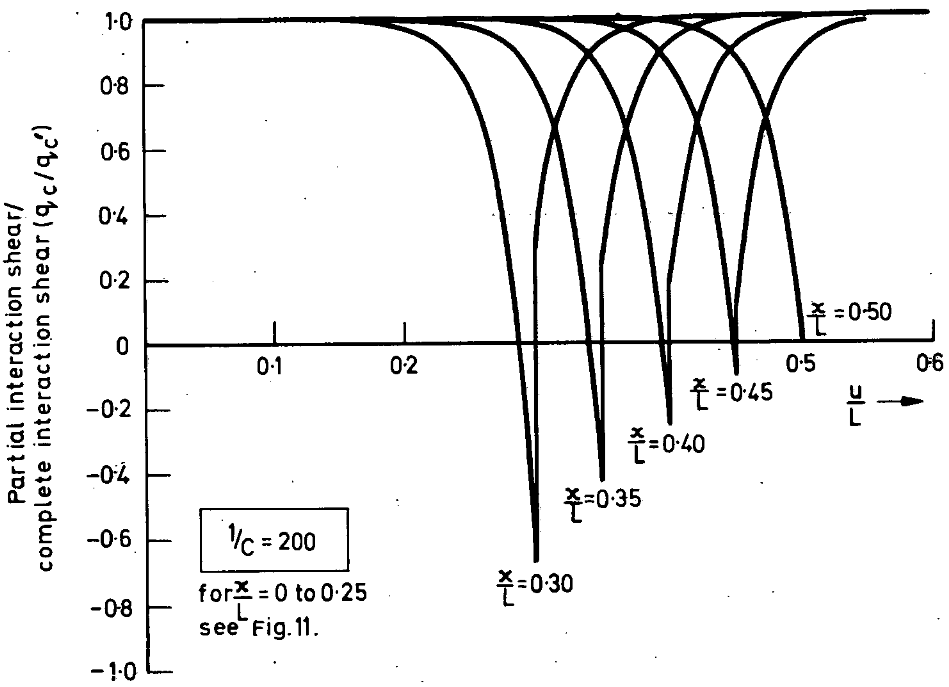


Fig. 10. INFLUENCE LINES FOR INTERFACE SHEAR WITH PARTIAL INTERACTION ($^1/_C$=200)

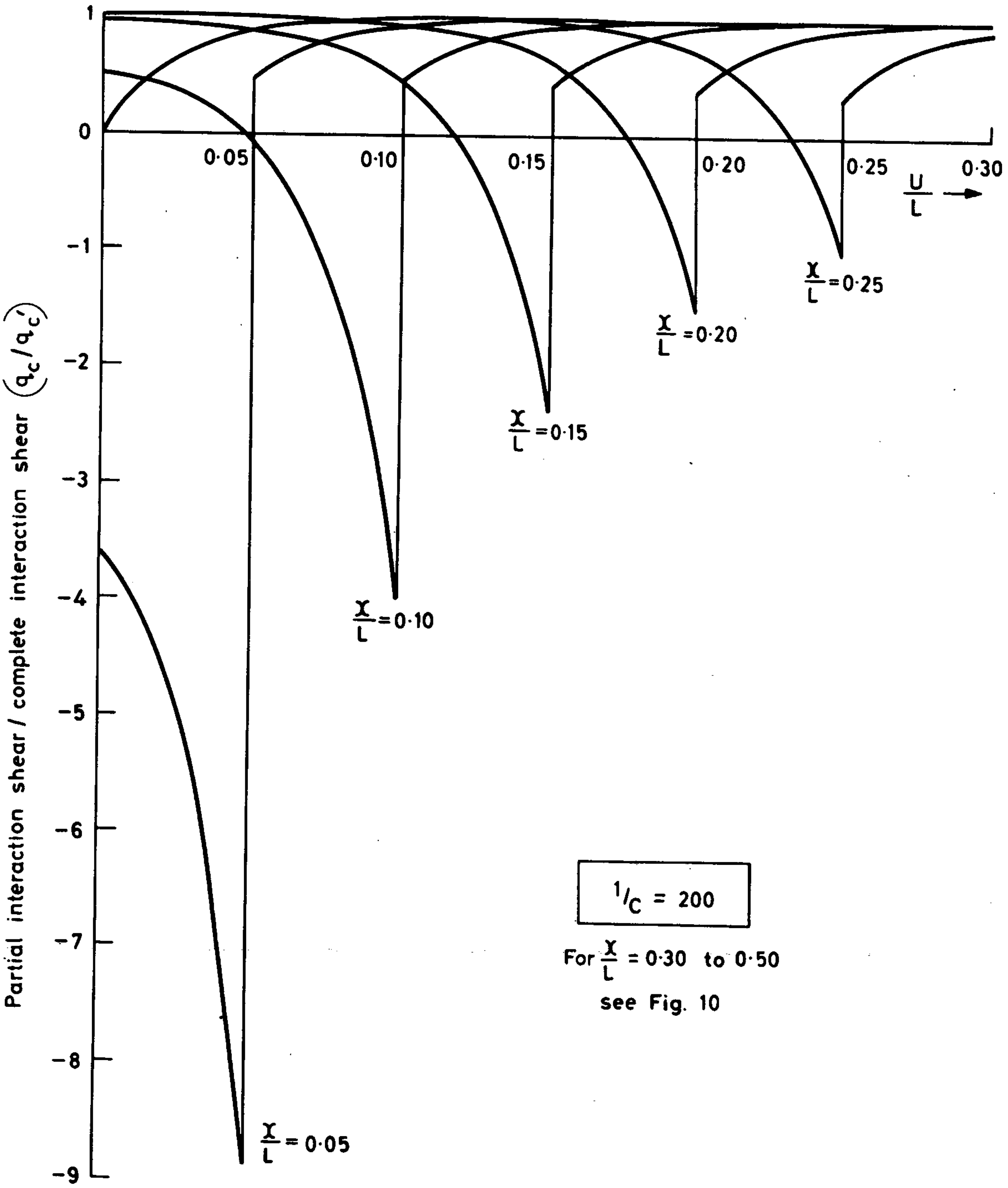


Fig. 11. INFLUENCE LINES FOR INTERFACE SHEAR WITH PARTIAL INTERACTION $(1/C = 200)$

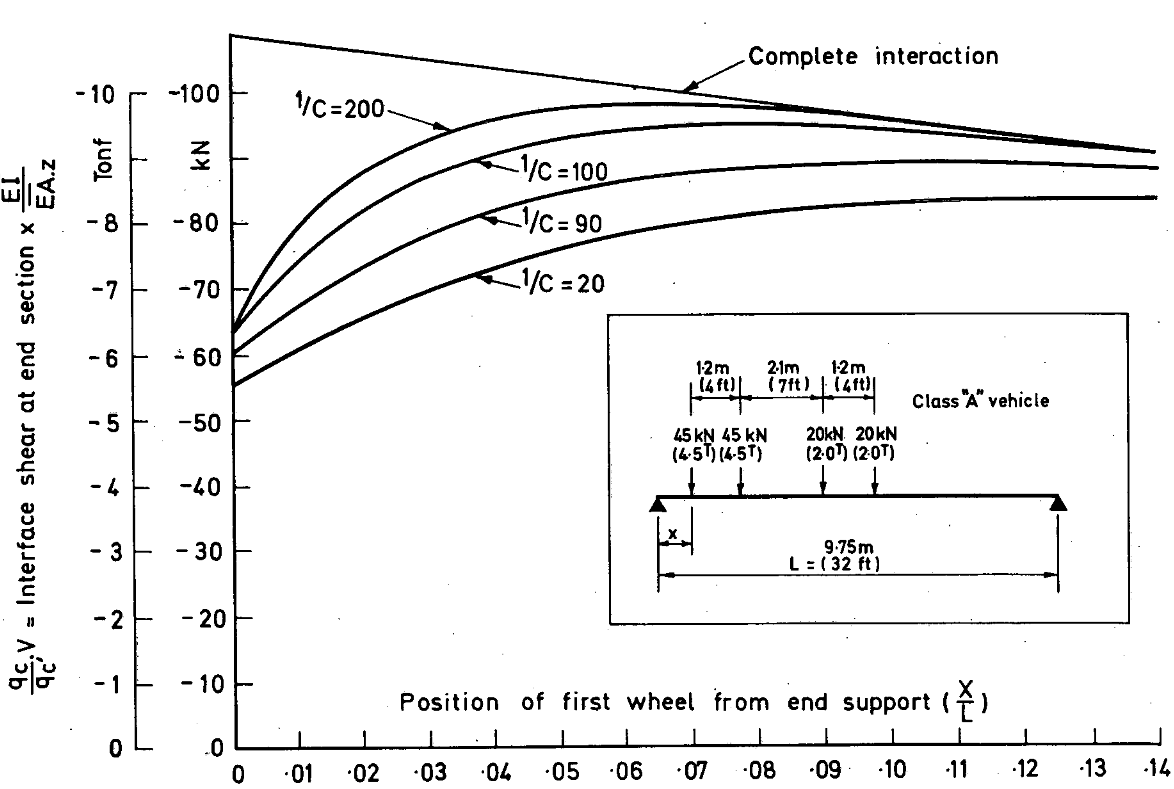


Fig.12. INTERFACE SHEAR AT END SECTION

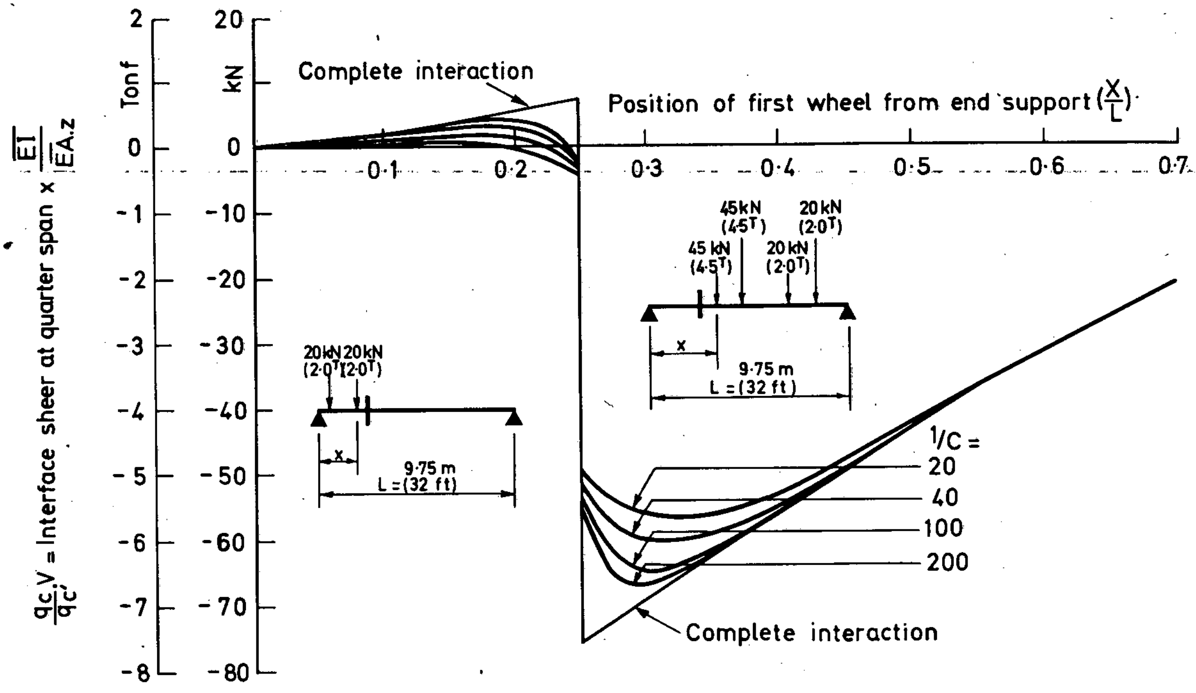


Fig.13. INTERFACE SHEAR AT QUARTER SPAN SECTION

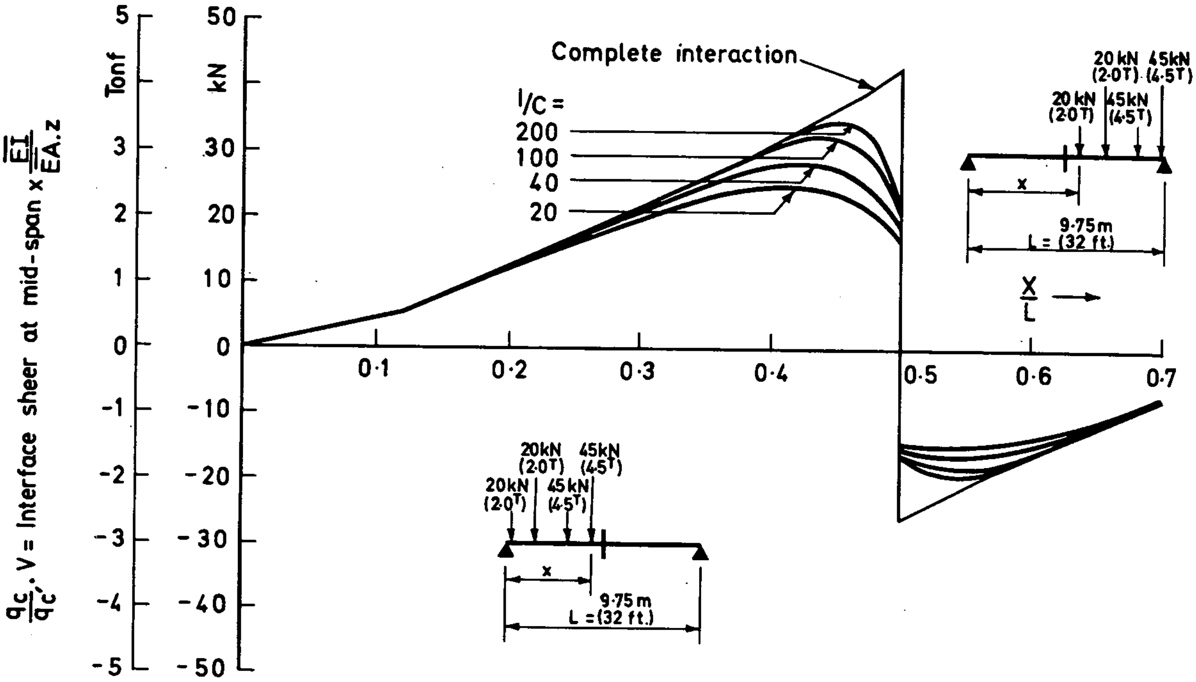
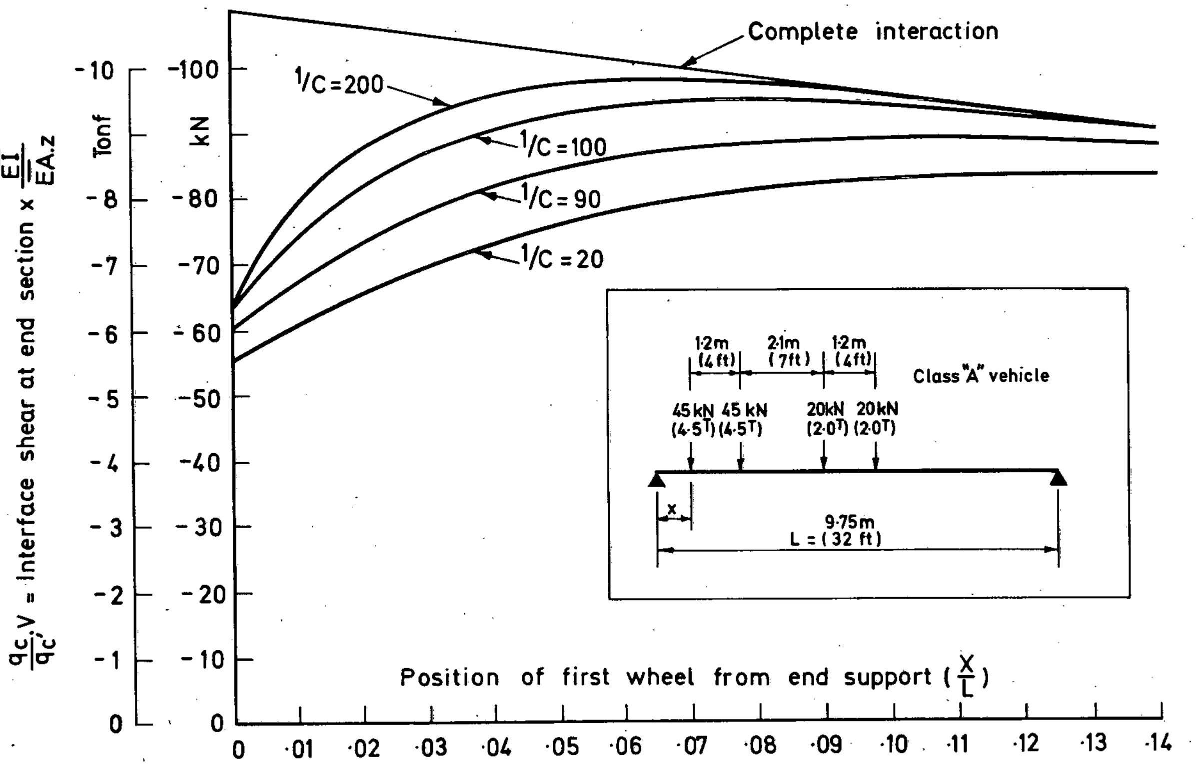


Fig.14. INTERFACE SHEAR AT MID-SPAN SECTION

Printed at the Road Research Laboratory, Crowthorne, Berkshire.



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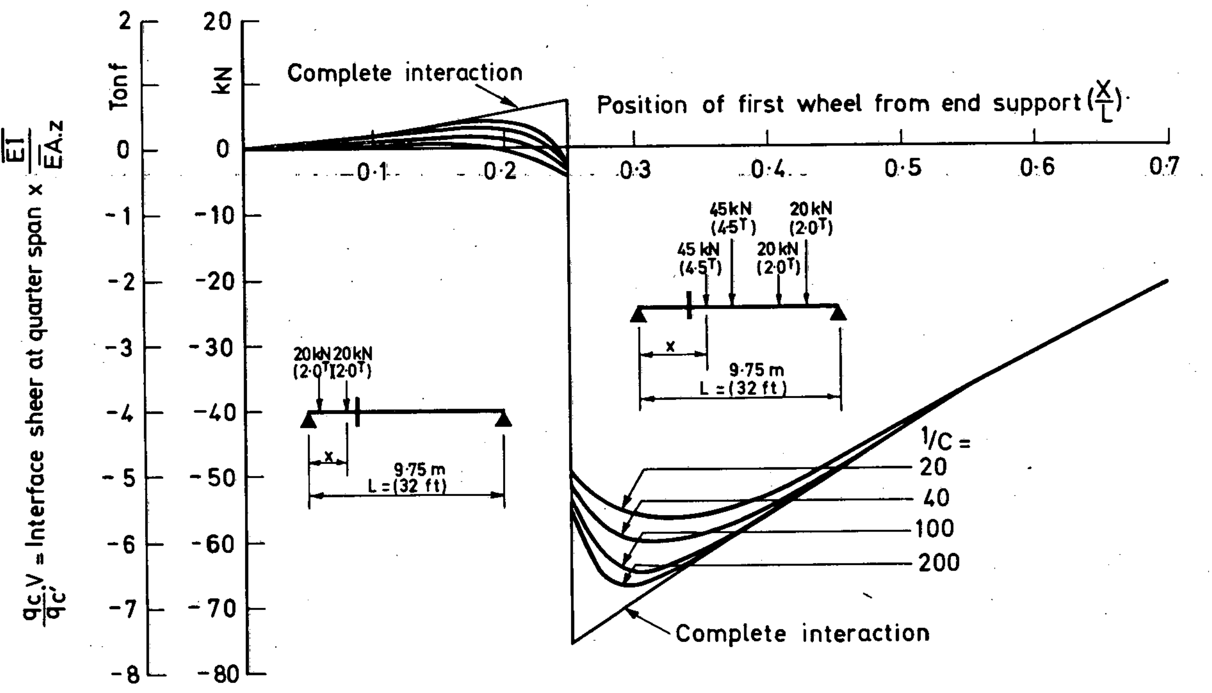

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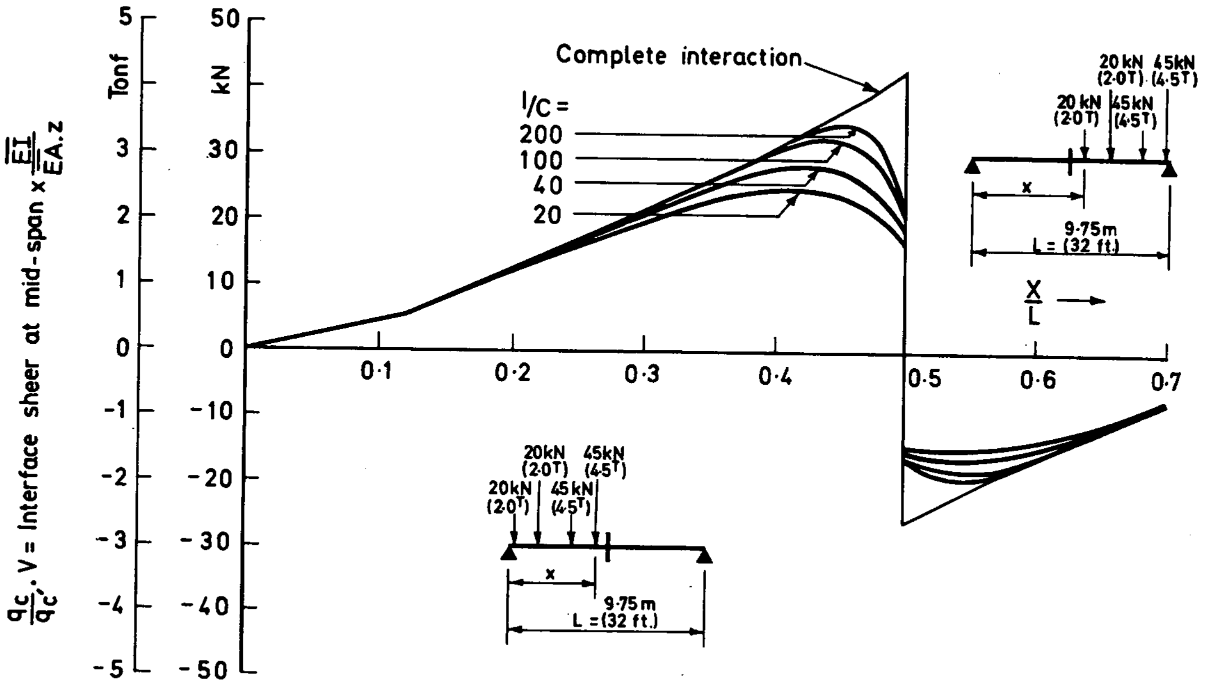

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