THE ESTIMATION OF FLOOD FLOWS FROM
NATURAL CATCHMENTS

by

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This report has been reproduced by permission of the Controller of HMSO. Extracts from the text may be reproduced, except for commercial purposes, provided the source is acknowledged.
A description is given of the development and testing of a method of estimating flood flows from natural catchments at desired return periods. The work is based on rainfall and run-off data recorded on a continual basis for several years at four natural catchments adjacent to motorways and supplemented by results from an additional natural catchment which was studied in an earlier investigation.

The initial phase of the study was a statistical analysis of the rainfall and stream flow data to determine the significant factors from which an equation relating a "time of concentration" to certain catchment features was deduced. Peak rates of flow may then be calculated using the Bilham rainfall formula, the average annual rainfall of the catchment and the catchment area. The method was based on data from natural catchments with an underlying stratum of clay or boulder clay and, when tested on similar gauged catchments, gave reasonable agreement between the calculated and observed flows.

A preliminary study of other natural catchments of greater or lesser permeability was made and the difference between the observed and calculated flows was explained by considering the catchment properties. Hence a modifying factor is incorporated in the equations. The method is primarily for the calculation of peak rates of flow and is not intended for the determination of complete hydrographs.

1. INTRODUCTION

The hydrological behaviour of small natural catchments bordering motorways has been studied by the Transport and Road Research Laboratory for several years with the aim of producing a method for the accurate estimation of flood flows from ungauged catchments. Such a method will facilitate the calculation of optimum dimensions of culverts and small bridges when planning new motorways or other roads. Present methods of flood flow prediction are often unreliable, over-estimating the flow at some required period leading to the design of a culvert of excessive capacity with consequent economic penalties.

Four small catchments were instrumented to provide continuous records of rainfall and run-off. Results from these catchments and another studied earlier were analysed to determine the effect of the catchment features in converting storm rainfall to run-off. This led to the prediction of peak rates of run-off from a knowledge of the rainfall and the catchment parameters.

This method has reasonable accuracy of estimation as well as simplicity of use so that its further development to produce entire hydrographs was not considered. While a hydrograph approach appears to give a less empirical and more complete solution, the assumptions involved in the solution of the various equations for ungauged catchments remove many of the apparent advantages. Also, the lack of suitable rainfall profiles for design purposes further reduces the applicability of a hydrograph method. However, the theory of hydrographs is examined to show that the important conclusions have a theoretical foundation. This is important because such a foundation enables the results to be extrapolated with confidence.
2. DESCRIPTION OF EXPERIMENTAL CATCHMENTS

Rainfall and run-off data were obtained from five experimental catchments. Full descriptions of these have been previously published\(^1,2\) and the salient features of each catchment are given below.

2.1 Harlow, Essex

This catchment, Fig. 1, with an area of 21.3 km\(^2\), is now built-up but when studied was mainly farmland. The ground was generally undulating, rising from about 38 m A.O.D. at the outfall to about 110 m A.O.D. at the upstream boundary. The underlying strata consist of Eocene London Clay covered by a small area of glacial sand and gravel near the outfall and by boulder clay* on the higher ground.

2.2 Flore, Northamptonshire

This catchment, Fig. 2, is adjacent to the M1 motorway and contains both pasture and some cultivated land, the area being 6.81 km\(^2\). The outfall is at about 77 m A.O.D. and the upstream boundary ranges in height from 122 m to 146 m A.O.D. Jurassic deposits of Upper Lias clay, silty clay and Marlstone Rock Bed cover nearly all the area, drift deposits being very small.

2.3 Upper Flore, Northamptonshire

This catchment is an upstream section of the previous one and has an area of 2.77 km\(^2\). The land rises from the outfall at about 87 m A.O.D. to the upstream boundary at about 146 m A.O.D. The geology is the same as that of Flore and Fig. 2 also shows this catchment.

2.4 Sandbach, Cheshire

This catchment, Fig. 3, has an area of 4.37 km\(^2\) and is bordered by the M6 motorway. Only moderate slopes are present, the land rising about 28 m from the outfall (at about 50 m A.O.D.) to the upstream boundary. Most of the land is pasture and the underlying strata are boulder clay over Triassic Keuper Marl.

2.5 Claughton, Lancashire

This catchment, Fig. 4, is bordered by the M6 motorway and has an area of 3.15 km\(^2\). The land rises steeply from the outfall at about 21 m A.O.D. to the upstream boundary at about 116 m A.O.D. The land is mainly pasture, the underlying stratum being boulder clay. This deposit is over Permian sandstone in the vicinity of the outfall and over Carboniferous Bowland Shales in the upstream area.

2.6 Summary

A range of conditions is covered, the areas varying from 2.77 km\(^2\) (Upper Flore) to 21.3 km\(^2\) (Harlow) and differing in shape also - Sandbach having a high length to width ratio which is opposite to that of Harlow. The catchment slopes vary, Claughton being relatively steep while Sandbach has only a moderate slope. The total stream length of each catchment is different, Claughton having more stream tributaries than Sandbach or Upper Flore. The areas have dissimilar average annual rainfalls; Flore, Upper Flore and Harlow receiving an average annual rainfall of about 650 mm, Sandbach about 825 mm and Claughton about 1100 mm. A common feature of the catchments is that they are all relatively impermeable, containing soils based on clays or boulder clay. Thus, the catchments are typical of many of those likely to be encountered in the United Kingdom.

* The term “boulder clay” is used here and elsewhere in the Report to mean stones of undefined size in a clay matrix.
3. INSTRUMENTATION

3.1 Rainfall

Three Dines tilting-siphon rainfall recorders were installed around the perimeter of each of the catchments as indicated in the previous figures and were each fitted with a clockwork strip chart mechanism (Plate 1) designed by the Transport and Road Research Laboratory. This unit records for about 30 days at a chart speed of 25 mm/h. In addition, a non-siphoning rainfall recorder using magnetic tape as a recording medium was installed in each catchment at a later date. This equipment, Plate 2, is described in detail elsewhere and was used to check the Dines' rainfall recorders.

Dines tilting-siphon rain-gauges were also used at Harlow but in this instance, were each fitted with a chart recorder of 24 hours duration.

3.2 Streamflow

A recorder house and concrete standing-wave flume were installed at the outfall of each catchment as indicated in Figs 1-4, the Sandbach flume being illustrated in Plate 3. The flumes were constructed to specifications prepared by the Hydraulics Research Station to provide maximum accuracy at both high and low rates of flow. The theoretical streamflow calibrations were confirmed over a large range of flow rates by current-meter measurements. Streamflow information was recorded continuously by clockwork chart mechanisms, Plate 4. The mechanisms at Flore, Sandbach and Cloughton had a chart speed of 25 mm/h with a duration of about 30 days while the Harlow recorder had a chart speed of about 3 mm/h with a duration of 8 days.

The Upper Flore streamflow recording station, Plate 5, was a simpler structure installed later. It consisted of a marine plywood plate containing a rectangular notch placed across the stream, the depth of water being recorded by a similar system to that of the non-siphoning rain-gauge.

3.3 Period of record

Data were collected for a 4 to 6 year period from all the catchments except Upper Flore which was studied for about 18 months. During the cattle foot and mouth epidemic of 1967-68 about 6 months of record were lost at Flore over the winter period when access restrictions prevented maintenance visits.

Some rainfall information was lost in winter when the Dines floatchambers froze, although the weekly addition of anti-freeze gave some protection. Occasional failures of the chart recorders occurred but produced little loss of record as weekly maintenance visits were made by the local River Authorities.

4. AN OUTLINE OF THE THEORY OF HYDROGRAPHS

In a real catchment the translation of rainfall into run-off is extremely complex and can be treated theoretically only by making simplifications of the various processes involved. The following analysis shows that using the observed delay between rainfall and run-off as a catchment model the important factors governing the relation between rainfall and peak rate of flow can be deduced using only mathematical simplifications. The experimental evidence confirms that these are justified.

As run-off lags rainfall for most catchments it is common to treat a catchment as a series of reservoirs whose characteristics are adjusted to give an observed hydrograph. For any given storm it is possible to derive the characteristics of a single reservoir that gives the observed hydrograph. As this will include rainfall losses its characteristics will vary from storm to storm.
Let the characteristics of this single reservoir be defined generally by:

\[ s = f(q) \] .......................................... (1)

where \( s \) is the quantity of water stored and \( q \) is the rate of flow at the outfall.

The inflow rate and outflow rate of the catchment will be given by the reservoir equation

\[ \frac{ds}{dt} - q = m \] ................................... (2)

where \( i \) is the inflow rate and \( t \) is the time, it being assumed that the units are homogeneous.

The solution of equation (2) is given by

\[ t - \int_0^t \frac{\mu \ i \ dt}{f'(q)} \]...

where \( \mu = \text{Exp} \left( \int_{-}^{t} \frac{dt}{f'(q)} \right) \)

Equation (3) cannot be solved unless both \( f'(q) \) and \( i \) are expressed in terms of \( t \). The instantaneous inflow rate \( i \) can be expressed as a mean, \( \bar{i} \), and a deviation, \( i(t) \) from the mean, i.e.

\[ i = \bar{i} + i(t) \] .................................... (4)

where by definition \( \int_0^t i(t) \ dt = 0 \)

Most hydrographs that result from a single rainfall event rise steadily to a peak, i.e., they are, in mathematical terms, continuous, monotonic functions. This is also true of storage-discharge relationships. Thus the stream flow can be represented by a power series in \( t \), time, and the reservoir characteristic \( s = f(q) \) by a power series in \( q \), i.e.

\[ q = \psi(t) = \sum_{j=1}^{n} b_j t^j \] .................................... (4a)

\[ s = f(q) = \sum_{j=1}^{n} k_j q^j \] .................................... (4b)

The substitutions give after some algebraic manipulation (Appendix 1) the solution of equation (3) as

\[ \frac{t}{k_1} + \bar{i} \left[ g_2 t^2 + g_3 t^3 + \cdots \right] + \frac{1}{\mu} \int_{0}^{t} \frac{\mu(t) \ dt}{f'(q)} \] .................................... (5)

where the coefficients \( g \) are the appropriate combinations of \( k_j \) and \( b_j \).
Now by definition $\int_0^1 i(t) dt = 0$ and the function $\mu/i'$ is such that it tends to be small.

Thus $\frac{1}{\mu} \int_0^1 \mu i(t) dt$ can only be a small quantity. Also the coefficients $g_i$ in equation (5) are such that the contribution to $q$ given by $\int i(t) dt \sum g_i t^i$ is also small. Hence equation (5) can be written

$$q = \frac{\ddot{i}}{k_1} + \epsilon$$

where $\epsilon$ is a small quantity, and $k_1$ is the first coefficient in the series for $s$ (equation 4b). This coefficient has the dimension of time. Putting $k_1 = k$ and $it = r$ where $r$ is the rainfall quantity (expressed as a volume) gives

$$q = \frac{r}{k} + \epsilon$$

Equation 7 shows that peak rates of flow are proportional to rainfall quantity and that variations in rate of rainfall are unimportant. They become more important for small catchments with short times of concentration but such catchments will not often need be considered for culvert design. In equation (7) the rainfall is expressed as a volume but if it is expressed in its most usual form as a linear measure equation (7) becomes

$$q = \frac{a r}{k} + \epsilon$$

where $a$ is an area. This cannot exceed the gross area, $A$, of the catchment. The constant $k$ will tend to minimum value, $T$, when rainfall losses are zero or small. Thus, the straight line

$$q = \frac{A r}{T} + \epsilon$$

will be an upper limit of possible peak rates of flow and will be proportional to quantity of rainfall.

It will be seen later that the experimental results confirm the above analysis and so the assumptions made are reasonable. This means that the results can be extrapolated with a considerable degree of confidence.

It must not be assumed from equation (8) that if the rain continues for a long time that the flow will also continue to rise. Eventually, the outflow rate will equal the inflow rate when the rate of change of water stored becomes zero. When this happens the flow represented by equation (8) becomes constant. If there is another storm with a higher rainfall intensity and the values of $a$ and $k$ in equation (8) remain the same then the point at which $q$ becomes constant is higher than for the lower intensity. Equation (9) is the equation of the envelope of all possible flows and represents the highest flows that can occur for a catchment. Fig. 5 illustrates these points.

There are two further inferences that can be drawn from this analysis. Firstly, while $t \leq k$ the peak flow will occur at the end of the rainfall. Secondly, and more important, hydrographs for a given catchment will be similar in shape, assuming there is no second period of rainfall affecting the flow from the first. This is in agreement with Sherman’s Unit Hydrograph theory, the validity of which has been demonstrated many times.
5. EXAMINATION OF STORM DATA

5.1 Statistical analysis

A multiple regression analysis was carried out on data abstracted from the entire period of record of each catchment. The selected parameters were the amount of rainfall, the duration of rainfall, the mean intensity for the duration of rainfall, the initial and peak rates of run-off, the flood flow (i.e. the peak rate minus the initial rate of run-off) and the volume of storm run-off. The various flows are illustrated in Fig. 6.

For a given storm, each rain-gauge in a catchment recorded a similar amount and duration of rainfall so that arithmetic average values were used: the corresponding storm run-off amount was obtained from the appropriate hydrograph. Simple hydrographs were uncommon in winter months, a later hydrograph commencing before recession of an earlier one had been completed. Estimates of the entire recession curve and hence the total run-off amount were then made by assuming that the flow rate decreased exponentially⁴ although this is only true over a limited duration of the recession curve.

The results of the analysis are similar for all the catchments and confirm the expected relationship given by equation (8). The actual correlation coefficients for Sandbach are given in Table 1. Of particular interest is the strong correlation between flood flow and volume of run-off. This is important because for practical purposes it removes the necessity of having to estimate percentage run-offs (i.e. the ratio of run-off volume to rainfall volume) when estimating high flows. This is similar to one of the conclusions of a previous investigation into urban hydrology².

| TABLE 1 |
| Correlation coefficients of rainfall and run-off parameters for Sandbach (107 results) |
| (NS means not significant) |

<table>
<thead>
<tr>
<th>Initial rate of run-off</th>
<th>Peak rate of run-off</th>
<th>Flood flow</th>
<th>Run-off amount</th>
<th>Rainfall amount</th>
<th>Rainfall duration</th>
<th>Mean rainfall intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial rate of run-off</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak rate of run-off</td>
<td>0.291</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flood flow</td>
<td>0.160</td>
<td>0.991</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Run-off amount</td>
<td>0.135</td>
<td>0.907</td>
<td>0.916</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rainfall amount</td>
<td>-0.236</td>
<td>0.732</td>
<td>0.788</td>
<td>0.780</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Rainfall duration</td>
<td>0.135</td>
<td>0.484</td>
<td>0.480</td>
<td>0.618</td>
<td>0.521</td>
<td>1.000</td>
</tr>
<tr>
<td>Mean rainfall intensity</td>
<td>-0.292</td>
<td>0.067</td>
<td>0.111</td>
<td>-0.003</td>
<td>0.269</td>
<td>-0.320</td>
</tr>
</tbody>
</table>
5.2 Development of the relationship between rainfall and run-off

Graphs of flood flow against rainfall amount, e.g. Flore (Fig. 7), Claughton (Fig. 8), show that a given rainfall amount produces either a relatively high or low flood flow depending upon the month of occurrence and hence upon the prevailing soil moisture deficit. A given quantity of rain will produce a higher flood flow when the soil moisture deficit is low as in December, January and neighbouring months. If, however, both the rainfall intensity is low and the period of fall is longer than the time of concentration, then the flood flows will be similar in both summer and winter. These, however, are unimportant because the resulting flood flows are too low to be of interest in the design of most drainage works. For Flore and Harlow, the low soil moisture deficit extends from about November to February inclusive and from mid-September to February inclusive for Claughton and Sandbach. Several Claughton and Sandbach results show that the soil moisture deficit can be low for short periods in the other months also, so that a given rainfall amount produces a flood flow as high as if the rain had occurred in December or the neighbouring months.

It is important to note that for each experimental catchment, the events occurring under a low soil moisture deficit do not only produce high flood flows but also lie along a straight line following equation (8). Because the minimum soil moisture deficit occurs most frequently in winter the data produced under this condition are defined as “winter” data.

The winter data of each catchment were compared, each flood flow being first normalised by converting it to a run-off intensity or output rate, \( I_H \), by the equation:

\[
I_H = \frac{3.6 \ Q}{A}
\]

where

- \( I_H \) = run-off intensity (mm/h)
- \( Q \) = flood flow (m³/s)
- \( A \) = catchment area (km²)

The Harlow, Flore, Sandbach and Claughton winter results appear in Figs 9, 10, 11 and 12 respectively, and confirm the validity of equation (9).

A single straight line is a good fit to all the data of each catchment. This is in contrast to the sometimes stated view that the results should be represented by a family of straight lines, each of a different slope according to a particular rainfall intensity. The correlation coefficients are shown on the figures and are significant at the 0.1 per cent level. Also, the Claughton results show that this linear increase in run-off intensity with rainfall amount appears to be valid even when the run-off intensity approaches the rainfall intensity. The best fit lines do not pass through the origin (though by definition they should), the intercept being the quantity \( e \) in equation (8). It can be seen that this quantity is small, as deduced. Therefore, the largest peak flood flow arising from a given rainfall for a catchment can be given approximately by

\[
Q = \frac{Ar}{k}
\]

where \( A \) is the catchment area, \( r \) is the rainfall amount and \( k \) is a constant with the dimension of time. The line represented by equation 10 is the envelope of all possible flows for periods of rain up to the time of concentration.

If the rainfall period is such that the banks of the water course are overtopped, then the characteristics of the assumed reservoir change altering the value of \( k \) so that it increases. This means that the line represented by equation (10) must eventually become curved such that true values of the largest peak flows are less than those given by equation (10). Thus the equation will probably over-estimate the very largest peak flows. This point is discussed more fully in Section 7.
The constant $k$ is the coefficient of the first term in the series expansion of the storage function which represents the action of the catchment, equation (4b).

This equation can be expressed as

$$s = kq + k_2 q^2 + k_3 q^3 + \cdots$$

where $k_1 = k$. By suitably adjusting the coefficients $k_2, k_3, \text{etc}.$, $s$ becomes

$$s = kq + k'_2(kq)^2 + k'_3(kq)^3 + \cdots$$

where $k'_2 = k_2/k^2$, etc. As there is no need to differentiate between the coefficients of the series equation (10) can be written

$$s = kq + k_2(kq)^2 + k_3(kq)^3 + \cdots$$

The quantity $k$ is the "lagging" factor of the reservoir. As $k$ increases so does the delay between rainfall and run-off and the degree of smoothing.

Equation (10) is the upper limit of possible flood flows and in this case the value of $k$ is the lowest value that the lagging factor can take, assuming the catchment is behaving normally. This lowest value is akin to a time of concentration which is usually defined as the time required for run-off from the most remote part of a catchment to reach the outfall. Denoting the lowest value of $k$ as $T$ equation (10) becomes

$$Q = \frac{Ar}{T}$$

The actual values of $T$ for the experimental catchments can be deduced from Figs 9, 10, 11 and 12. This has the disadvantage that the high values of $Q$ required to define the curve are few because they are rare events. By manipulating the appropriate equations (Appendix 2) it is possible to derive values of $T$ by two methods using all the data. The values given by the three methods were similar and the average value is given in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Upper Flore</th>
<th>Flore</th>
<th>Sandbach</th>
<th>Claughton</th>
<th>Harlow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (km$^2$)</td>
<td>2.77</td>
<td>6.81</td>
<td>4.37</td>
<td>3.15</td>
<td>21.33</td>
</tr>
<tr>
<td>$F$</td>
<td>1.05</td>
<td>1.20</td>
<td>1.74</td>
<td>1.40</td>
<td>0.91</td>
</tr>
<tr>
<td>$V$</td>
<td>0.421</td>
<td>0.922</td>
<td>0.846</td>
<td>0.738</td>
<td>0.548</td>
</tr>
<tr>
<td>$E$ (km$^{-1}$)</td>
<td>0.92</td>
<td>1.16</td>
<td>1.04</td>
<td>4.42</td>
<td>1.28</td>
</tr>
<tr>
<td>$L$ (km)</td>
<td>1.75</td>
<td>3.10</td>
<td>3.63</td>
<td>2.49</td>
<td>4.20</td>
</tr>
<tr>
<td>$S_M$</td>
<td>1/24.1</td>
<td>1/43.0</td>
<td>1/135</td>
<td>1/21.1</td>
<td>1/44.0</td>
</tr>
<tr>
<td>$S_A$</td>
<td>1/19.2</td>
<td>1/32.0</td>
<td>1/124</td>
<td>1/19.6</td>
<td>1/38.5</td>
</tr>
<tr>
<td>$S_S$</td>
<td>1/35.2</td>
<td>1/52.5</td>
<td>1/131</td>
<td>1/25.9</td>
<td>1/61.5</td>
</tr>
<tr>
<td>$T$ (h)</td>
<td>13.1</td>
<td>16.7</td>
<td>27.5</td>
<td>12.4</td>
<td>23.7</td>
</tr>
<tr>
<td>$F_A$</td>
<td>-</td>
<td>0.448</td>
<td>0.836</td>
<td>1.056</td>
<td>0.516</td>
</tr>
<tr>
<td>$R_A$ (mm)</td>
<td>-</td>
<td>639</td>
<td>814</td>
<td>1113</td>
<td>683</td>
</tr>
</tbody>
</table>
6. THE TIME OF CONCENTRATION RELATED TO CATCHMENT FEATURES

In order to use equation (13) for the prediction of flood flows for ungauged catchments it is necessary to relate the time $T$ to the appropriate topographical features of the catchment. The value of $T$ must be a function of the rate of movement of water in the catchment and the distances travelled. At present it is not possible to derive values from purely theoretical considerations and so a statistical analysis must be used to determine the most significant catchment features and their relation to $T$. With only five experimental catchments, the number of characteristics that could be considered was limited and the following were selected as being those of major importance - area, shape, drainage density, length and slope, their values being given in Table 2.

The area, $A$, of each catchment was readily available while the shape was defined by a shape factor, $F$, expressed as $(\text{catchment length} / \sqrt{\text{catchment area}})$ which has been used elsewhere. While this factor gives an indication of the difference in shape of a catchment from that of a square, it does not take into account the distribution of area along the length of a catchment. Therefore an additional factor, the area variation, $V$, was also determined. The perpendicular bisector of the catchment length, $L$, (which is defined below) was extended to the boundaries to form two sub-areas. The ratio of the downstream to the upstream area is the factor, $V$. The drainage density, $E$, expressed as the total length of the stream network divided by the catchment area was also used. This term is imprecise due to the difficulty in deciding what constituted a stream or where it ended.

The catchment length, $L$, was measured from a map of scale 1:25,000 or 1:10,560 and is the plan distance of a line from the outfall to the upstream divide approximately along the centre of the catchment - minor irregularities in the catchment shape being ignored. The more irregular shapes of some of the catchments required the use of a cranked line, but, in general, the location of each line was not critical - various routes giving similar distances. (A transition curve connecting each limb of a cranked line was thought unnecessary). The routes used are indicated in Figs 1, 2, 3 and 4 for the experimental catchments. There is negligible difference between each plan distance and the corresponding overland distance as the horizontal slope angle is small.

Three definitions of catchment slope were examined, each being the ratio of the number of metres rise to the number of metres travelled. The median slope, $S_M$, and the mean slope, $S_A$, were found by placing a grid of equally-spaced lines over a 1:25,000 or 1:10,560 scale catchment map so that from 50 to 100 grid line inter-seCTIONS lay within the catchment. The minimum distance between the two adjacent contours at each inter-section was measured and converted to a slope. The values were ranked in order of magnitude and a graph of slope value against frequency of occurrence was prepared from which the median slope, $S_M$, was found. The mean slope, $S_A$, is the arithmetic mean of all the slopes. Another measure of slope used was the simple slope, $S_S$, which is the ratio of $Z$, the difference between the height of the outfall and the average height of the upstream divide, to the catchment length, $L$. The extent of each upstream divide is also shown in Figs 1 to 4 inclusive, the use of an average height being more meaningful than the particular height of the divide where intersected by the catchment length line.

The significance of the variables was tested by correlating each of them with the time of concentration. Since time equals distance divided by velocity and velocity is a function of slope, it is reasonable to expect that the best relationship between the variables will be of the form:

$$T = f(L^x/S^y, A, E, V, F)$$

a logarithmic relationship being assumed. It was found that the variables, $A$, $E$, $V$ and $F$ were not significant, only the length, $L$, and the slope (however defined) being important. Clearly, the predominant features of catchment length and slope outweigh any effects of catchment shape which was originally thought to be important since Upper Flore, Claggarton and Harlow are of irregular form. However, it is possible that more irregularly shaped catchments might have values of $T$ that are dependent on their shapes.
SS has the highest correlation (0.94) of the three measures of slope while the correlation coefficient of length, L, is 0.85. These values are significant at least at the 10 per cent level. A regression analysis using these two significant variables leads to an equation of the form:

\[ T = 2.48 \frac{L^{0.39}}{S_{S}^{0.39}} \]  

Equation (14) may be rewritten in a more convenient form:

\[ T = 2.48(LN)^{0.39} \]  

where \( N \) is a 'slope number', and

\[ N = S_{S}^{-1} = \frac{L}{Z} \]

The multiple correlation coefficient of L and SS with T is 0.98 which is highly significant.

Both L and SS in equation (14) are simple measures of length and slope respectively in that they ignore any meanderings of the stream in a catchment. Thus, it seems that a better relationship could be derived from the actual stream length and some better slope measure. In practice, however, difficulties arise because a stream usually has several tributaries and choosing the correct one is not easy. Also the measurement of stream lengths is affected by the scale of the catchment map, a smaller map not displaying all the irregularities of the stream path. Consequently, it was not practical to derive an equation that gives better results than equation (15).

Equation (15) can be compared with the Bransby Williams formula which is often used to calculate the time of concentration in the United Kingdom. This method was originally developed for catchments in India and the times of concentration produced are very short. For catchments of approximately circular shape, the equation is:

\[ T = \left( \frac{A_{M}}{H} \right)^{0.2} \]

where \( T \) = time of concentration (h)

\( A_{M} \) = catchment area (miles\(^2\))

\( H \) = average number of feet fall per 100 feet from the edge of the watershed to the outfall.

A correction is used for irregularly shaped catchments. The power 0.2 in the equation reduces the effect of \( A_{M} \) or \( H \) so that comparatively large catchments have times of concentration by this equation of from 1 to 2 hours. For example, Flore has a time of concentration by this equation of only 1 hour compared to a measured value of about 17 hours. This formula cannot therefore be recommended for use in the United Kingdom.

7. THE ESTIMATION OF FLOW RATES FOR REQUIRED RETURN PERIODS

For the hydraulic design of bridges and culverts and also for many other drainage structures the expected peak flood flow, \( Q \), that will be exceeded for a chosen probability, frequency or return period \( P(q > Q) \) is needed. If the catchment is ungauged the distribution of the flood flows, \( q \), is not known and hence the probability \( P(q > Q) \) cannot be calculated. The flood flow, \( q \), is a function of rainfall and so the distribution of \( q \) is inferred from that of the rainfall.
In the design of bridges and culverts it is usually only those values of flood flow that are in the higher region of the distribution of all flood flows that are of interest. These flows are those expressed by the linear equation (13). To derive \( P(q > Q) \) it is assumed that

\[
P(q > Q) = P(r > R)
\]

where \( R \) is the rainfall amount giving \( Q \) and \( r \) is any rainfall amount, i.e., it is assumed that rates of flow occur with the same probability or frequency as the rainfall that produces them.

If the appropriate rainfall amount, \( r \) in equation (13) is known that equation can be used to predict the required flood flow. Existing rainfall statistics only give probabilities as a function of both rainfall amount, \( r \), and the duration \( t \), giving rise to that amount, i.e.

\[
P(t \leq T) = f(R,T)
\]

where \( t \) is any duration. Thus, in order to deduce \( R \), \( T \) has to be chosen in addition to the probability \( P(t \leq T) \). This raises difficulties as there is no adequate definition of duration. In order to avoid a completely arbitrary choice a duration equal to the time \( T \) in equation (13) has been chosen because it can be calculated from the appropriate catchment parameters. The calculated flow resulting from this choice will be in error as the actual duration, however defined, could be different. This error can be corrected, subject to an assumption.

Suppose that for a given probability, \( P(r > R_A) \), the quantity of rainfall to give the required flow, \( Q_A \), is \( R_A \):

\[
Q_A = \frac{AR_A}{T}
\]

If the rainfall amount associated with the time of concentration, \( T \) for the same probability is \( R_B \),

\[
Q_B = \frac{AR_B}{T}
\]

i.e. \( Q_A = \frac{R_A}{R_B} \cdot \frac{AR_B}{T} = Q_B \cdot \frac{R_A}{R_B} \)

Thus knowing \( Q_B \) it is possible to calculate \( Q_A \) if \( R_A/R_B \) is also known.

The quantities \( R_A, R_B \) will each be contained in some distribution and hence

\[
P(r > R_A) = F(r), \quad F(r) \in R_A
\]

\[
P(r > R_B) = \psi(r), \quad \psi(r) \in R_B
\]

The probabilities \( P(r > R_A) \) and \( P(r > R_B) \) are to be equal and hence

\[
F(R_A) = \psi(R_B)
\]

The distributions \( F(r), \psi(r) \) will be of the same type but certain parameters, such as mean and variance, will have different values.
The distributions containing $R_A, R_B$ must have a range $0 \leq r < \infty$ and will be such that there will be more low values than high values. These are characteristics of exponential type distributions. If it is supposed that the distributions are exponential then

$$P(r > R_A) = e^{-\lambda_A R_A}$$

and

$$P(r > R_B) = e^{-\lambda_B R_B}$$

Where $\lambda_A, \lambda_B$ are the reciprocals of the means of the distributions. Therefore,

$$\lambda_A R_A = \lambda_B R_B$$

The distributions will have means and if it is supposed that they cover the same periods in years then

$$\lambda_A = \frac{1}{n R_A} \quad \text{and} \quad \lambda_B = \frac{1}{n R_B}$$

Where $\bar{R}_A, \bar{R}_B$ are annual average rainfalls and $n$ is the period in years. Therefore,

$$\frac{R_A}{R_B} = \frac{\bar{R}_A}{\bar{R}_B} = \frac{Q_A}{Q_B} \quad \ldots \quad \ldots \quad \ldots$$

(17)

and hence if the rainfall distributions are exponential or at least a reasonable approximation to exponential, the correcting factor $R_A/R_B$ will be proportional to annual average rainfall.

In the United Kingdom the Bilham formula$^{9,10}$ is most often used to express the relation between rainfall quantity, duration and probability. A series of flood flows, $Q_B$, corresponding to a range of probabilities were calculated for each experimental catchment using the formula and equation (13), taking the duration to be equal to the appropriate value of $T$. These flows were compared with the observed values, $Q_A$, for the same probabilities. If equation (17) is true then the ratio $Q_A/Q_B$ will be constant, though in practice it will rise to a constant value because at the high probability levels the observed values of $Q_A$ will be too low because there is insufficient rain to wet fully the ground. Fig. 13 shows the variation of $Q_A/Q_B$ with probability expressed as a return period for the four experimental catchments. It can be seen that each ratio does rise to a constant value and that their order is the same as that of their respective annual average rainfalls. The scatter at the longer return periods is due to the comparative shortness of the period of observation. Fig. 14 shows the variation of the observed values of $Q_A/Q_B$ with annual rainfall. Despite there being only four catchments the expected linearity is clearly demonstrated. The correlation coefficient is 0.96 which is significant at the 5 per cent level.

It seems reasonable to suppose that the distributions of rainfall are at least a good approximation to an exponential distribution and that calculated value of $Q_B$ can be converted to actual value, $Q_A$, by multiplying by the ratio of the annual average rainfalls.
According to equation (17) it should be possible to deduce the correction factor $\frac{R_A}{R_B}$ from a knowledge of the annual average rainfall of the area typified by the rainfall statistics leading the calculation of the rainfall quantity $R_B$. As it is difficult to decide what this area might be, the most practical way of predicting $\frac{R_A}{R_B}$ for the United Kingdom is to use the observed correlation shown in Fig. 14. This has been used successfully in Section 8 of this Report which discusses the application of the proposed design method to a series of gauged catchments chosen from the Surface Water Year Book.

In theory the variation of the factor $\frac{R_A}{R_B}$ with annual average rainfall is a straight line through the origin. In practice there will be some curvature close to origin because this region represents the less wet areas which will tend to have a greater proportion of flows, $q$, occurring when the ground is dry, i.e. the same reason for the curvatures of the lines shown in Fig. 13. This is equivalent to saying that $P(q > Q_A) \neq P(r > R_A)$, the equality of which is one of the assumptions made in the derivation of $Q_A$. The non-linearity is represented by the constant of the "best-fit" line through the points shown in Fig. 14.

To summarise the actual peak flow, $Q_A$, for some required probability, $P(q > Q_A)$, or return period is given by

$$Q_A = Q_B \frac{R_A}{R_B} = \frac{A}{T} R_B \frac{R_A}{R_B} \ldots \ldots \ldots \ldots \ldots (18)$$

where $R_B$ is the quantity of rainfall given by

$$P(q > Q_A) = P(r > R_B) = f(R_B, T)$$

In the United Kingdom $R_B$ will usually be given by the Bilham formula in which case $\frac{R_A}{R_B}$ is best calculated from the equation of the best fit line through the points shown in Fig. 14, i.e.

$$F_A = \frac{R_A}{R_B} = 0.00127 R_A - 0.321 \ldots \ldots \ldots \ldots \ldots (19)$$

The value of $Q_A$ calculated above neglects base flow. This can be added to $Q_A$ if it is known. If it is neglected the error is unlikely to exceed 10 per cent at a return period of once in four years. For longer return periods the error will be even less.

The estimation of flow rates in the above manner assumes that (a) the actual duration of rainfall is less than or equal to $T$, the lowest lag or concentration time, and (b) the water is not overflowing its banks. In estimating flood flow rates for design purposes the limitations implied by the above assumptions are not important because equation (13) is the upper limit of possible flows. An exception to this can occur if the design probability is such that the banks of the waterway are over-topped. This does not affect the validity of equation (13) but it means that the value of $T$ will change, because the shape of the storage characteristic, $s = f(q)$ changes. In practice there will be a discontinuity in the curve so that it cannot readily be represented by a single series.

It is possible to infer the effect of overtopping of the banks. In equation (3) the limits of integration can be for any period and by the rules of integration can be split into any number of separate, but consecutive, periods. This means that for a time $t$, where $t > T$, equation 3 can be written

$$q = f(t) \int_0^t \psi(t)dt = f(T) \int_0^T \psi(t)dt + f(t) \int_T^t \psi(t)dt.$$
Now \( f(T) \int_0^T \psi(t) \, dt \) is the peak flow as if the rain stopped at time \( T \), i.e. that flow \( q_T \), given by equation (13). Thus

\[
q = q_T + \int_0^T f(t) \psi(t) \, dt
\]

The contribution \( f(t) \int_0^T \psi(t) \, dt \) is that part occurring after the over-topping. When this happens there will be a big increase in water stored with only a small increase in flow. Hence at the point of discontinuity the slope of the storage characteristic curve will increase, i.e., the value of \( k \) for this part of the curve will be greater than that for the first part of the curve. Thus the contribution \( f(t) \int_0^T \psi(t) \, dt \) will be less than if it was supposed that the initial value of \( k \) held for that part of the curve and after the discontinuity, i.e., by supposing that equation (13) is true for all values of \( r \). Thus the peak flow cannot exceed that given by equation (13) for all values of rainfall quantity. Therefore, if the calculated flow is such that banks are over-topped there is no danger of under-estimating this flow. It is not possible, however, to say what is the degree of over-estimation. The error involved is much less than that given by the traditional methods of calculating peak flood flows.

8. THE APPLICATION OF THE METHOD TO OTHER GAUGED CATCHMENTS

The effectiveness of the proposed method was assessed by testing it on other gauged natural catchments having long-term records of the highest peak rate of run-off for each ‘water’ year, (October to September). Unfortunately, the number of catchments available for testing is restricted for several reasons. Some flow-recording stations are of inadequate capacity while others have inaccurate calibrations or insufficient periods of record. There are few catchments similar in size to the experimental catchments and larger ones were selected, some with an average rainfall exceeding that of Claughton.

The annual maximum series of the recorded peak flows was plotted on Gumbel\(^{11}\) probability paper and the technique described in this reference was used to calculate the best-fit line from which the observed peak flows were determined for return periods within the range 2 years to the maximum duration of the flow record. The method described by Gringorten\(^{12}\) was employed to calculate the 95 per cent confidence limits of the recorded flow results.

When comparing calculated flows with these observed flows an allowance has to be made because the calculated flows are based on a partial-duration series while the observed flows are based on an annual maximum series. The two series differ increasingly for return periods below 10 years, but the known relation between the two series enables them to be compared. Return periods of the partial-duration series, equivalent to the 2 year, 3 year, 4 year, etc., return periods of the annual maximum series, are listed in Table 3. Also, when comparing the observed and calculated flows, the average annual rainfall for the period of flow record was used and not the long-term average annual rainfall for the period 1916-1950. However, when determining a flow for some design period, the long-term average must be used because firstly, the actual annual rainfall for any year cannot be predicted, and secondly, the design period itself is not fixed, it being any period containing that number of years.

The catchments tested are named after the appropriate flow-gauging stations and are described in Table 4. The details consist of the Hydrometric Area Number (as found in the Surface Water Year Book\(^{13}\)), drift and solid geology, area, area variation, average annual rainfall, slope number, length and time of concentration (as calculated from equation (15)). The agreement of the calculated flood flows, \( Q_C \), with the observed peak flows is also included in the form of the ratio (calculated flow/observed flow) for return periods of 2 years up to the maximum duration of the observed flow record.
<table>
<thead>
<tr>
<th>Annual maximum series</th>
<th>Partial-duration series</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.417</td>
</tr>
<tr>
<td>1.5</td>
<td>0.910</td>
</tr>
<tr>
<td>2.0</td>
<td>1.443</td>
</tr>
<tr>
<td>3.0</td>
<td>2.466</td>
</tr>
<tr>
<td>4.0</td>
<td>3.476</td>
</tr>
<tr>
<td>5.0</td>
<td>4.481</td>
</tr>
<tr>
<td>10.0</td>
<td>9.491</td>
</tr>
<tr>
<td>15.0</td>
<td>14.49</td>
</tr>
<tr>
<td>20.0</td>
<td>19.50</td>
</tr>
<tr>
<td>30.0</td>
<td>29.50</td>
</tr>
<tr>
<td>40.0</td>
<td>39.50</td>
</tr>
<tr>
<td>50.0</td>
<td>49.50</td>
</tr>
<tr>
<td>100.0</td>
<td>99.50</td>
</tr>
</tbody>
</table>

Initially, the method was applied to catchments where the underlying stratum nearest the surface is clay or boulder clay, as in the experimental catchments. These test catchments are Newton, Harper’s Brook, Glem, Ash Bourne, Irfon, Wye at Pant Mawr and Wye at Cefn Brwyn. Maps of five of these catchments appear in Figs 15 to 19 inclusive, the catchment lengths and upstream and downstream sub-areas being shown also. The calculated flows are in good agreement with the observed flows at all return periods for all seven catchments as shown in the Table. Considering the longest return periods which are of main interest, Irfon has the worst agreement, the 30 year flow ratio being 1.30, a difference from unity which is not excessive. The flow ratios at the minimum return period of 2 years are also satisfactory, the worst ratio being 1.21 for Harper’s Brook.

The results are also shown in Figs 20 to 24 inclusive for the same five catchments as previously illustrated. As would be expected from the discussion of the flow ratios, the calculated flows lie close to the best-fit line of recorded flows and hence are generally within the 95 per cent confidence limits of the recorded flow results.
<table>
<thead>
<tr>
<th>Hydro-</th>
<th>Catchment</th>
<th>Drift and solid geology</th>
<th>Area (km²)</th>
<th>Area variation</th>
<th>Average annual rainfall (mm)</th>
<th>Slope number</th>
<th>Length (Km)</th>
<th>Time of concentration (h)</th>
<th>Calculated flow/observed flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>R.Inzioin near Loch of Lintrathen</td>
<td>Glacial sand and gravel and some boulder clay over Extrusive Igneous highly-fractured Schistose Grit.</td>
<td>24.7</td>
<td>0.518</td>
<td>1064</td>
<td>25.9</td>
<td>7.79</td>
<td>19.9</td>
<td>1.94 1.79 1.84 1.92 -</td>
</tr>
<tr>
<td>15</td>
<td>R.Newton at Newton</td>
<td>Mainly boulder clay and fine clay over Extrusive Igneous schists - much less glacial sand and gravel present than in previous catchment.</td>
<td>15.9</td>
<td>0.930</td>
<td>1196</td>
<td>23.1</td>
<td>11.5</td>
<td>22.1</td>
<td>1.10 1.10 1.16 1.23 -</td>
</tr>
<tr>
<td>21</td>
<td>Fruid Water at Fruid</td>
<td>Boulder clay covers 62 per cent of area, remainder is thin soil over Silurian Greywacke grits &amp; shale bands.</td>
<td>23.8</td>
<td>0.449</td>
<td>1625</td>
<td>22.4</td>
<td>9.88</td>
<td>20.6</td>
<td>1.04 1.14 1.25 1.36 -</td>
</tr>
<tr>
<td>24</td>
<td>Bedburn Beck at Bedburn</td>
<td>Mainly shallow soil over Carboniferous Millstone Grit with about 25 per cent of the area covered by boulder clay.</td>
<td>74.2</td>
<td>0.587</td>
<td>868</td>
<td>34.6</td>
<td>12.8</td>
<td>27.0</td>
<td>0.89 0.93 0.99 - -</td>
</tr>
<tr>
<td>24</td>
<td>Rookhope Burn at Eastgate</td>
<td>As above but with only about 5 per cent by area of boulder clay present.</td>
<td>36.6</td>
<td>1.13</td>
<td>1220</td>
<td>34.2</td>
<td>10.9</td>
<td>25.3</td>
<td>0.73 0.83 0.92 - -</td>
</tr>
<tr>
<td>Catchment</td>
<td>Drift and solid geology</td>
<td>Calculated flow/observed flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harper's Brook at Old Mill Bridge</td>
<td>Mainly boulder clay (71 per cent by area) with underlying Jurassic clay, limestone and marl exposed near the stream.</td>
<td>1.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R. Box at Police Ford</td>
<td>70 per cent of area is boulder clay, remainder is glacial sand and gravel. Underlying stratum is Cretaceous chalk.</td>
<td>1.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R. Glenn at Gemford Mill</td>
<td>88 per cent of area is boulder clay, remainder is glacial sand and gravel. Underlying stratum is Cretaceous chalk.</td>
<td>1.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R. Bbourne at Hadlow Bridge</td>
<td>Few drift deposits. Cretaceous clay (47 per cent of area) and chalk are present.</td>
<td>1.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ash Bourne at Hammer Wood Bridge</td>
<td>Lower Cretaceous deposits are present. Clay covers 60 per cent of the area and contains many sand and silt seams. Hovering the remaining area.</td>
<td>1.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Time of concentration (h) | 63.3 | 20.9 | 186 | 627 | 1.41 |
| Length (Km) | 11.2 | 15.0 | 54.5 | 55.9 | 1.00 |
| Area variation (mm) | 1.12 | 11.5 | 31.2 | 46.4 | 146.4 |
| Area (Km²) | 55.2 | 88.7 | 0.621 | 1.00 | 18.4 |

**TABLE 4: Catchment details (continued)**
## TABLE 4

Catchment details (continued)

<table>
<thead>
<tr>
<th>Hydrologic area No.</th>
<th>Catchment</th>
<th>Drift and solid geology</th>
<th>Area (km²)</th>
<th>Area variation</th>
<th>Average annual rainfall (mm)</th>
<th>Slope number</th>
<th>Length (km)</th>
<th>Time of concentration (h)</th>
<th>2 year return period</th>
<th>5 year return period</th>
<th>10 year return period</th>
<th>20 year return period</th>
<th>30 year return period</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>R. Fowey at Trekeivesteps</td>
<td>Mainly shallow soil over igneous granite. A few small areas of peat present.</td>
<td>36.8</td>
<td>1.54</td>
<td>1656</td>
<td>102</td>
<td>13.7</td>
<td>42.4</td>
<td>0.91</td>
<td>0.96</td>
<td>1.02</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>55</td>
<td>R.Irfon at Abernant</td>
<td>Boulder clay over Ordovician and Silurian shales and sandstone bands.</td>
<td>72.6</td>
<td>1.07</td>
<td>1801</td>
<td>47.7</td>
<td>17.6</td>
<td>34.7</td>
<td>0.92</td>
<td>1.02</td>
<td>1.11</td>
<td>1.22</td>
<td>1.30</td>
</tr>
<tr>
<td>55</td>
<td>R.Wye at Cefn Brwyn</td>
<td>Boulder clay over Silurian shales and mudstones.</td>
<td>10.5</td>
<td>0.784</td>
<td>2437</td>
<td>12.4</td>
<td>4.40</td>
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<td>Shallow soil over Carboniferous Millstone Grit.</td>
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<td>Crosdale Brook at Crosdale Flume</td>
<td>Shallow soil over mainly Carboniferous Millstone Grit with a small area of Carboniferous shales near the outfall.</td>
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**NOTE:** The catchments containing significant areas of permeable deposits at the surface have the impermeable areas and the flow ratios based on them shown in brackets.
The seven catchments cover a wide range of areas, area variations, average annual rainfalls, lengths and slopes. The good agreement between the calculated and observed flows demonstrates that the method can be used for any catchments where the deposit nearest the surface (including any drift deposit) is clay or boulder clay.

It is obviously advantageous to be able to apply the method to catchments with other types of strata. Catchments with a shallow soil of about a metre or less in depth overlying rock might be expected to produce flows greater than those calculated due to the lack of storage in the catchments. (However there is, in fact, little moisture change at depth below about one metre for catchments having an underlying stratum of clay, so that this supposition might not be true.) Also excessive flows might be expected for catchments having more permeable strata nearest the surface. The application of the method to catchments of each type was therefore examined.

Six catchments of the former type are Rookhope Burn, Croasdale Brook, Fowey, Bedburn Beck, Fruid Water and Bottoms Beck. The first three contain negligible boulder clay and only a shallow soil overlying rock. For these catchments the method gives reasonable flow estimates, these being 73, 95 and 91 per cent respectively of the actual at the 2 year return period - the average being 86 per cent. This scatter might be due partly to the relatively short record period of 10 years.

These values are not greatly different from those obtained for the previous catchments of clay or boulder clay, only the Rookhope Burn value being lower. Therefore, there is little evidence to suggest that modification of the method is necessary when applying it to catchments having only a shallow soil overlying rock.

Fruid Water has about 60 per cent of its area covered by boulder clay, the remainder being shallow soil over rock and as might be expected, the flow ratios are near unity - the 2-year ratio being 1.04. Bottoms Beck, with no boulder clay present, has much lower flow ratios - the 2 year ratio being 0.55, i.e., the calculated flows are too low. However, only this catchment is appreciably afforested, nearly all the area being covered by a plantation of conifers. Investigations showed that the afforested area has an extensive network of ditches which will convey water rapidly to the outfall and cause the catchment to respond to rainfall more rapidly than grassed catchments. A more suitable catchment is Bedburn Beck which has mainly a shallow soil overlying rock. It contains coniferous forest also but only over about 20 per cent of the area so that higher flow ratios are produced, i.e., 0.89 at the 2 year return period. Clearly, other heavily afforested catchments need to be examined before the applicability of the method to them can be ascertained.

Box, Bourne and Inzion are examples of catchments with large areas of permeable strata near the surface. Box, which has boulder clay over 70 per cent of its area and glacial sand and gravel over the remaining area, has a flow ratio of 2.16 at a 2 year return period. Bourne has 53 per cent of its area covered by chalk and sand while Inzion has a large area of sand and gravel but of unknown extent and these catchments also have large flow ratios of 1.77 and 1.94 respectively at the 2-year return period. The ratios at longer return periods for all three catchments are similar to their respective 2-year return period values. Modification of the calculated flows by some coefficient is probably an over-simplification of the factors involved and additional permeable catchments need to be studied. Lacking this information, an approximate correction can be made which consists of substituting the area, \( A_L \), of the catchment that is covered by impermeable deposits, e.g., clay, in place of the entire catchment area, \( A \), when calculating \( Q_A \) from equation (18). However, the time of concentration is based on the entire catchment area as previously described. The basis for this correction is that, firstly, the permeable area is assumed to make a negligible contribution to the peak rate of run-off. Secondly, the impermeable stratum is assumed to lie along the catchment length line from the outfall to the upstream divide so that the time of concentration is unaffected by the revised catchment area. This correction reduces the Bourne and Box flow ratios at the 2-year return period to 0.83 and 1.5 respectively. Inadequate knowledge of the geology of Inzion prevents the determination of its modified flow ratio.
9. CONCLUSIONS AND SUMMARY OF THE METHOD

A flood estimation method has been developed for the determination of peak rates of flow from natural catchments but it is not intended for the estimation of complete hydrographs. The method is relatively simple and resembles the "rational" or "Lloyd-Davis" method which, ignoring any constants of conversion, is normally expressed as:

\[ Q = CAI \]  \hspace{1cm} (20)

where \( C \) = coefficient considered to be dependent on the catchment characteristics. The term in equation (18) equivalent to the rational coefficient, \( C \), is dependent on the average annual rainfall and not on the catchment characteristics. Nevertheless, it is still of the rational type and such equations have the advantage that their physical meaning is reasonably clear, a balance between the input and output rates of a catchment being made. Early rational equations give excessive flows due to the use of a time of concentration of insufficient duration as described in Section 6 while selection of an appropriate value of \( C \) is difficult. The method described in this Report has none of these faults and is applicable to a wide range of catchments.

The technique is intended primarily for natural catchments with an underlying deposit nearest the surface of clay or boulder clay - hence drift deposits must be considered. For the seven catchments tested which had such deposits, the average calculated flow exceeded the actual flow by only about 15 per cent at the 20-year return period. Adaptation of the method to natural catchments having other deposits is proposed, also. Further research is needed to check if this method can be applied to heavily afforested catchments.

The equations necessary to determine the peak rates of flow at required return periods are, in order of usage:

\[ T = 2.48 (LN)^{0.39} \]  \hspace{1cm} (21)

where,

- \( T \) = lagtime or time of concentration when all the catchment area is contributing to the flood flow (h).
- \( L \) = catchment length from outfall to upstream divide, being measured approximately along the middle of the catchment. A cranked line may be necessary if the shape is sufficiently irregular (km).
- \( N \) = dimensionless slope number equal to the ratio, \( L/Z \), where \( Z \) is the rise from the outfall to the average height of the upstream divide in kilometres.

The time of concentration can be conveniently obtained from Fig. 25 consisting of a graph of \( T \) against the product, LN, for durations extending from 10 hours to 60 hours.

The expected rainfall \( R_B \) (mm), is found from the Bilham rainfall formula for the duration, \( T \), and the selected return period, \( Y \), in years. The formula is held to be valid for durations of up to 48 hours and good agreement between calculated and observed flows has been shown in this Report for catchments having times of concentration of up to 63 hours. Table 5 gives the Bilham rainfall intensity, \( R_B/T \), for a range of return periods and durations, the latter starting at 10 hours.

The corresponding calculated flood flow, which approximates to the peak rate of run-off is calculated from:

\[ Q_C = FAAR_B/3.6T \]  \hspace{1cm} (22)
where \( A \) = catchment area (km\(^2\))

\[
F_A = \text{annual rainfall factor (dimensionless)}
\]

\[
= 0.00127 \bar{R}_A - 0.321 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)
\]

and \( \bar{R}_A \) = average annual rainfall (mm).

The constant 3.6 has been introduced to account for the differing dimensions of the variables. Equation (22) is based on data from catchments of average annual rainfall ranging from about 640 mm to 1110 mm but has been shown to be valid for average annual rainfalls of up to 2440 mm.

Provisionally, the method is also applicable, without any modification, to catchments having a shallow soil overlying rock. For use on catchments containing a significant area of permeable strata (e.g., sand, gravel, or chalk) nearest the surface, equation (2) is modified by substituting the area of the catchment, \( A_L \) (km\(^2\)) that is covered by impermeable deposits, e.g., clay, for the catchment area, \( A \), giving:

\[
Q_C = F_A A_L \frac{R_B}{3.6T} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)
\]

10. ACKNOWLEDGEMENTS

The authorities that supplied catchment information are listed below and their co-operation is gratefully acknowledged. Apologies are due to any organisation that may have been omitted from the list.

- Cornwall River Authority
- Dee and Clwyd River Authority
- East of Scotland Water Board
- Essex River Authority
- Forestry Commission
- Fylde Water Board
- Institute of Hydrology
- Kent River Authority
- Lancashire River Authority
- Mersey and Weaver River Authority
- Meteorological Office
- Northumbrian River Authority
- South-East of Scotland Water Board
- Sussex River Authority
- Tweed River Purification Board
- Water Resources Board
- Welland and Nene River Authority
- Wye River Authority
- Yorkshire Ouse and Hull River Authority
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TABLE 5
Values of $R_g/T$ from the Bilham formula
11. REFERENCES


12. SYMBOLS

\(a\) = any area

\(A\) = catchment area (km\(^2\))

\(A_C\) = area of catchment contributing to the flood flow at a rate equal to the rainfall intensity (km\(^2\))

\(A_M\) = catchment area (miles\(^2\))

\(A_L\) = area of catchment covered by impermeable deposits (km\(^2\))

\(b_j(j=1,n)\) = Coefficients in flow power series

\(C\) = rational equation coefficient (dimensionless)

\(D\) = depth of water over area, \(A\) (mm)

\(D_C\) = depth of water over area, \(A_C\) (mm)

\(E\) = drainage density (km\(^{-1}\))

\(F\) = shape factor (dimensionless)

\(F_A\) = annual rainfall factor (dimensionless)

\(g_j(j=2,n)\) = coefficients in a power series

\(H\) = catchment slope defined as average number of feet fall per 100 feet from edge of watershed to the outfall (dimensionless)

\(i\) = any inflow rate

\(I\) = rainfall intensity (mm/h)

\(I_H\) = run-off intensity (mm/h)

\(k\) = lagging factor with dimension time

\(k_j(j=1,n)\) = coefficients in reservoir characteristic power series

\(L\) = catchment length from outfall to upstream divide (km)

\(m\) = slope of best-fit line equation (2) (h\(^{-1}\))

\(n\) = period of years

\(N\) = catchment slope number equal to \(L/Z\) (dimensionless)

\(P_Y\) = average annual rainfall (mm)

\(q\) = any outflow rate or flood flow rate

\(Q\) = flood flow (m\(^3\)/s)
$Q_A$ = observed flood flow from the run-off records (m$^3$/s)

$Q_B$ = flood flow derived from rainfall statistics (m$^3$/s)

$Q_C$ = calculated flood flow, approximating to the actual peak rate of flow at a given return period (m$^3$/s)

$r$ = any rainfall amount

$R$ = rainfall amount giving rise to a flood flow $Q$ (mm)

$R_A$ = rainfall amount giving rise to a flood flow $Q_A$ (mm)

$R_B$ = rainfall amount giving rise to a flood flow $Q_B$ (mm)

$R_{A}, R_{B}$ = annual average rainfalls

$s$ = any quantity of stored water

$S_A$ = average catchment slope equal to the proper fraction, metres rise/metres travelled (dimensionless)

$S_M$ = median catchment slope, defined as above (dimensionless)

$S_S$ = simple catchment slope equal to the proper fraction, $Z/L$ (dimensionless)

$t$ = any time

$T$ = time of concentration - being the time required for run-off from the most remote part of a catchment to reach the outfall (h)

$T_F$ = rainfall duration (h)

$V$ = area variation factor (dimensionless)

$v$ = run-off volume

$Y$ = return period (years)

$Z$ = rise in height from outfall to averaged height of upstream divide (km)

$\varepsilon$ = a small quantity

$\mu$ = an integrating factor

$\lambda_{A,B}$ = means of rainfall distributions
13. APPENDIX 1

THE DERIVATION OF THE FLOOD-FLOW RELATIONSHIP

The rate of flow, \( q \), after a time \( t \) is given by equation (3), i.e.

\[
q = \frac{1}{\mu} \int_0^t \frac{\mu_i \, dt}{f'(q)} + \frac{1}{\mu} \int_0^t \frac{\mu_i(t) \, dt}{f'(q)} \quad \ldots \quad \ldots \quad \ldots \quad (3)
\]

where \( \mu = \exp \int_0^t \frac{dt}{f'(q)} \)

and \( f'(q) = \frac{d(f(q))}{ds} \)

and \( s = f(q) \)

Put \( s = f(q) = k_1 q + k_2 q^2 + k_3 q^3 + \ldots \)

\[
= \sum_{j=1}^{n} k_j q^j \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4a)
\]

and \( q = (t) = b_1 t + b_2 t^2 + b_3 t^3 + \ldots \)

\[
= \sum_{j=1}^{n} b_j t^j \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4b)
\]

By definition the series are convergent for the required range. This being so then all manipulations using the series will lead to series which are also convergent.

Differentiating (4a) gives

\[
f'(q) = k_1 + \sum_{j=2}^{n} jk_j q^{j-1}
\]

Substituting (4b) for \( q \) gives

\[
f'(q) = k_1 + \sum_{j=2}^{n} jk_j \left( \sum_{j=1}^{n} b_j t^j \right)^{j-1} \quad \ldots \quad \ldots \quad \ldots \quad (25)
\]
In the following manipulations the complexity of the coefficients increases rapidly and consequently only the first one or two are worked out completely.

Expanding equation (25)

\[ f'(q) = k_1 + 2k_2 b_1 t + (2k_2 b_2 + 3k_3 b_1^2) t^2 + c_3 t^3 \quad ++ \]

Put \( c_2 = (2k_2 b_2 + 3k_3 b_1^2) \)

giving \( f'(q) = k_1 + 2k_2 b_1 t + c_2 t^2 + c_3 t^3 \quad +++ \)

where \( c_3 \) is the appropriate combination of \( k_j, b_j \)

Therefore

\[ \frac{1}{f'(q)} = \frac{1}{k_1} - \frac{2k_2 b_1 t}{k_1^2} + t^2 \left( \frac{4k_2 b_1^2 - k_1 c_2}{k_1^3} \right) \quad +++ \quad (26) \]

Now \( \mu = \exp \left( \int \frac{dt}{f'(q)} \right) \). Hence integrating (26)

\[ \int \frac{dt}{f'(q)} = \frac{t}{k_1} - \frac{2k_2 b_1 t^2}{2k_1^2} + t^3 \left( \frac{4k_2 b_1^2 - k_1 c_2}{3k_1^2} \right) \quad +++ \]

Replacing the exponential function by its series gives

\[ \mu = 1 + \frac{t}{k_1} - \frac{(2k_2 b_1 - 1)}{2} \frac{t^2}{k_1^2} + \frac{e_3 t^3}{k_1^3} \quad +++ \quad (27) \]

Thus

\[ \frac{\mu / f'(q)}{t} = \frac{1}{k_1} - (2k_2 b_1 - 1) \frac{t}{k_1} + f_2 \frac{t^2}{k_1^3} \quad +++ \]

giving

\[ \int_0^t \frac{\mu dt}{f'(q)} = \frac{t}{k_1} - \frac{t^2}{2k_1^2} \left( 2k_2 b_1 - 1 \right) + f_2 \frac{t^3}{3k_1^3} \quad +++ \]

Therefore

\[ \frac{1}{\mu} \int_0^t \frac{\mu dt}{f'(q)} = \frac{t}{k_1} - \frac{t^2}{k_1^2} \left( \frac{1 + 2k_2 b_1}{2} \right) + \frac{g_1 t^3}{k_1^3} \quad +++ \]
giving

\[ \int_0^t \frac{\mu dt}{\Gamma'(k)} = \frac{it}{k_1} - i \left[ g_2 \frac{t^2}{k_1^2} + g_3 \frac{t^3}{k_1^3} + \cdots \right] \]

where is equation (4).

It can be seen that coefficient \( g_2 \) is given by

\[ g_2 = \frac{1 + 2k_2b_1}{2} \]

It can be shown that all the coefficients \( g \) have the form

\[ g_j = (-1)^j \frac{(1 + h_j)}{j!} \quad j = 2, 3, \ldots, n \quad \cdots \quad \cdots \quad \cdots \quad (28) \]

where \( h_j \) is the appropriate combination of the coefficients \( a_j \) and \( b_j \).

The contribution of the neglected terms is likely to be greatest when \( t = k_1 \). When this happens

\[ \frac{1}{i} \left[ g_2 \frac{t^2}{k_1^2} + g_3 \frac{t^3}{k_1^3} + \cdots \right] \]

\[ = \frac{1}{i} \left[ \left( \frac{1}{2!} - \frac{1}{3!} - \frac{1}{4!} + \cdots \right) + \sum_{j=2}^{n} \frac{n}{j^2} - i h_j \right] \]

\[ = \frac{1}{i} \left[ e^{-1} + \frac{\beta}{j=2} - (-1)^j h_j \right] \]

This gives some indication of the shape of the envelope containing possible flows. However, this has little practical value because there cannot be a maximum flow for a catchment. If rainfall increases enough the water course will overflow altering the shape of the storage characteristic and hence increasing the value of the lag or concentration time.
14. APPENDIX 2

ADDITIONAL DERIVATIONS OF LAG OR CONCENTRATION TIME

Consider rain falling on a given catchment, area A, with a mean intensity $\bar{I}$ for a time $t \leq k$. Then by equation (13) the peak run-off will be

$$q = \frac{A \bar{I} t}{k}$$

neglecting the small quantity $e$.

This rate of flow can also be considered to be generated by the same rain falling on a smaller catchment for a time such that $k = t$,

i.e. $$q = \frac{a \bar{I} t}{t} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$ (29)

This can be written

$$t = k$$

$$a < A$$

or

$$\frac{a \bar{I} t}{t} = \frac{A \bar{I} t}{k}$$

or

$$a = \frac{A t}{k}$$

As $k$ is the shortest lag or concentration time

$$a = \frac{A t}{T} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$ (30)

Thus any flow from an area $A$ can be considered as a maximum flow from a smaller area, $a$. Relationships of this type have been described elsewhere\textsuperscript{2,6}.

The reduced area, $a$, can be found from recorded values using equation (29) and can be plotted against rainfall duration, $t$. The ratio $A/T$ can then be found from the slope of the line. Fig. 25 shows the data for Sandbach. This derivation of $T$ is, of course, little different from those given by the slopes of Figs 6, 7, 8 and 9, but it does give a different weight to the variables.

A more independent method can be deduced from the volume of run-off.

Suppose it rains for a time $t$. Then the volume of run-off in that time will be

$$v = \int_0^t q \, dt$$

29
Now by equation (13)

\[ q = \frac{a \bar{t}}{k} \]

which gives

\[ v = \int_0^t \frac{a \bar{t}}{k} \, dt \]

\[ = \frac{a \bar{t}^2}{2k} \]

\[ = \frac{a \bar{r} \bar{t}}{2k} \quad \text{where} \quad \bar{t} = \bar{r} \]

i.e. \[ \frac{v}{a} = \frac{\bar{r} \bar{t}}{2k} \]

The constant \( k \) is the lag or concentration time, \( T \), giving

\[ \frac{v}{a} = \frac{\bar{r} \bar{t}}{2T} \]

Thus if the volume of run-off under the rising limb of each hydrograph is divided by the catchment area, the value of this ratio should give a straight line of slope \( \frac{\bar{r}}{T} \) when plotted against the product \( \bar{r} \bar{t}/2 \). This was done for each catchment and the lag or concentration times given by the slopes were similar to those determined by the other methods. The results for Sandbach are shown in Fig. 26.
Fig. 1. THE HARLOW CATCHMENT
Fig. 2. THE FLORE AND UPPER FLORE CATCHMENTS
Rainfall recorder
Streamflow recorder
Catchment length

Fig. 3. THE SANDBACH CATCHMENT
• Rainfall recorder
• Streamflow recorder

m = Catchment length

0.25
0.5 (mile)
0.5
1.0 (km)

Fig. 4. THE CLAUGHTON CATCHMENT
Fig. 6. Definitions of initial rate of flow, peak rate of flow and flood flow.
Fig. 7. FLOOD FLOW AGAINST RAINFALL AMOUNT (FLORE)
Fig. 9. RUN-OFF INTENSITY AGAINST RAINFALL AMOUNT (HARLOW)
Slope = 1/10 h$^{-1}$
Correlation coefficient = 0.925

Fig. 10. RUN-OFF INTENSITY AGAINST RAINFALL AMOUNT (FLORE)
Fig. 11. Run-off intensity against rainfall amount (Sandbach)

Slope = 1/25.4 h^{-1}
Correlation coefficient = 0.937

Rainfall intensity (mm/h)
- 0-1
- 1-2
- 2-3
- 3-4
- 4-5
- 5-6
- 6-7
- 7-8
- 15-16
Average ratios $\frac{Q_{ACT}}{Q_{BL}}$ over period indicated:

- Claughton 1.056
- Sandbach 0.836
- Flore 0.448
- Harlow 0.516

**Fig. 13** $\frac{Q_{ACT}}{Q_{BL}}$ **AGAINST RETURN PERIOD FOR ALL CATCHMENTS**
Fig. 16. $F_A$ AGAINST AVERAGE ANNUAL RAINFALL OF THE CATCHMENTS
Boundary between sub-areas

Outfall

Fig. 15. THE RIVER NEWTON AT NEWTON CATCHMENT
Fig. 18. THE RIVER WYE AT CEFN BRWYN CATCHMENT
Fig. 19. THE RIVER WYE AT PANT MAWR CATCHMENT
Fig. 20. CALCULATED AND OBSERVED MINIMUM FLOWS - RIVER NEWTON AT NEWTON CATCHMENT
Fig. 21. CALCULATED AND OBSERVED MINIMUM FLOW - HARPER'S BROOK AT OLD MILL BRIDGE CATCHMENT
Fig. 22. CALCULATED AND OBSERVED MINIMUM FLOWS - RIVER GLEM AT GLENSFORD MILL CATCHMENT
Fig. 23. CALCULATED AND OBSERVED MINIMUM FLOWS - RIVER WYE AT CEFN BRWYN CATCHMENT
Fig. 24. CALCULATED AND OBSERVED MINIMUM FLOWS - RIVER WYE AT PANT MAWR CATCHMENT
Fig. 26. AREA–TIME DIAGRAM (SANDBACH)
Fig. 27. DEPTH ν/α Against RT/2 (SANDBACH)

\[ T = 27.5 \text{ h} \]

Correlation coefficient = 0.913
PLATE 4

Interior of Claughton flume recorder house showing chart recorder
The estimation of flood flows from natural catchments: C P YOUNG, BSc and J PRUDHOE, BSc, PhD: Department of the Environment, TRRL Report LR 565: Crowthorne, 1973 (Transport and Road Research Laboratory). A description is given of the development and testing of a method of estimating flood flows from natural catchments at desired return periods. The work is based on rainfall and run-off data recorded on a continual basis for several years at four natural catchments adjacent to motorways and supplemented by results from an additional natural catchment which was studied in an earlier investigation.

The initial phase of the study was a statistical analysis of the rainfall and stream flow data to determine the significant factors from which an equation relating a "time of concentration" to certain catchment features was deduced. Peak rates of flow may then be calculated using the Bilham rainfall formula, the average annual rainfall of the catchment and the catchment area. The method was based on data from natural catchments with an underlying stratum of clay or boulder clay and, when tested on similar gauged catchments, gave reasonable agreement between the calculated and observed flows.

A preliminary study of other natural catchments of greater or lesser permeability was made and the difference between the observed and calculated flows was explained by considering the catchment properties. Hence a modifying factor is incorporated in the equations. The method is primarily for the calculation of peak rates of flow and is not intended for the determination of complete hydrographs.