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A SUGGESTED METHOD OF RELIABILITY ANALYSIS FOR EARTH RETAINING STRUCTURES

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FOREWORD

This report must be seen in the context of the considerable controversy and uncertainty surrounding the possible adoption of limit state methods of design in the United Kingdom in ground engineering at the present time. Limit state design methods have been put forward in BS 5400 and the Department of Transport has adopted this method of design for bridges on motorways and trunk roads. Thus at the present time the design of bridge superstructures is based on limit state design methods while their foundations are designed using a permissible stress/ lumped factor of safety approach.

Because of the current uncertainty on how best to apply limit state design methods in ground engineering the laboratory has commissioned research on three different approaches to the problem. This report, on the use of reliability analysis, represents one such approach. Another involves the use of fuzzy-sets and research is being undertaken on this by Dr D I Blockley at the University of Bristol (Blockley 1980). At the University of Cambridge Mr M D Bolton is tackling the problem using a design value approach.

At this relatively early stage all three approaches are already providing valuable insights into the many approaches to limit state design. Critical examination of the various approaches highlights strengths and weaknesses and identifies anomalous behaviour and this enables more soundly based decisions to be made on the content and form of any limit state design methods proposed for use in ground engineering.

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G N Smith

ABSTRACT

A new method for determining β, the reliability index, (and hence the probability of failure) of an earth retaining structure, has been evolved.

The method employs a standard of mathematics no higher than that used in the deterministic evaluation of a factor of safety.

Although the method has not been tested by any rigorous mathematic treatment it has been numerically checked by Monte Carlo simulations.

Using the proposed method values of β have been obtained for several design problems. These values have been found to be very close to values of β obtained from Monte Carlo simulations of the problems (generally using not less than 10,000 iterations).

Because of the simplicity of the method it is hoped that it will prove helpful to civil engineers interested in the idea of using probability theory to assist in their design work.

1 INTRODUCTION

1.1 Limit state design

The ultimate limit state of a structure is defined as its state at collapse.

Such a collapse marks the actual end of the structure but there can often be other limits that mark the end of its useful life, such as excessive deflections, cracking, unacceptable vibration levels, fire damage, etc. These limits are referred to as serviceability limit states.

It is now recognised that there can be occasions when one or more of the serviceability limit states of a structure is more critical than its ultimate limit state.

The principle of limit state design is that, instead of simply designing for the ultimate limit state, all relevant serviceability limit states are examined and the most critical one determines the design.

1.2 The Factor of Safety

The suitability of a structure to withstand a particular loading is generally expressed in terms of its factor of safety, F.

This approach is satisfactory provided the design values of the parameters used in the determination of F are realistic. However there are recorded examples of geotechnical structures where the calculated factor of safety exceeded 1.0 and yet failure occurred. In some cases, particularly clay slopes, it was found that the properties of the soil had changed from the time they had been measured. These slope failures were therefore due to faulty design which used incorrect soil strength parameters. With other failures the predicted F values were most likely in error because of the selection of wrong design values from the test data available.

The possibility of selecting a wrong design value in soil calculations is seen to be quite high when it is appreciated that the maximum and minimum values of normally distributed variables are more or less equal to their mean values $\pm$ three times their standard deviations.

Consider, for example, the angle of shearing resistance, ϕ, of a natural sand deposit. Various workers Lumb (1966), Schultze (1972), have determined that this variable tends to follow a normal probability distribution with a coefficient of variation of some 5 to 15 per cent. If such a sand has a mean peak ϕ value of 35° with a standard deviation of 2° then its full range of possible peak ϕ values extends from 29° to 41°. The value selected from this range as the design value of ϕ will determine the value of $\tan\phi$ and therefore the value of F.

Generally the mean value of a parameter is used in design calculations but there are occasions when the choice of the minimum value is considered more prudent. A third value that might be selected for a parameter is its characteristic value, suggested in CP110, (1972), for reinforced concrete design. The code defines the characteristic strength as the value of the cube strength of the concrete, the yield or proof stress of the reinforcement, or the ultimate load of a prestressing tendon below which not more than 5 per cent of the test results will fall.

For a parameter with a normal distribution:—

Characteristic strength = Mean value − 1.645 × standard deviation

Characteristic load = Mean value + 1.645 × standard deviation

The draft standard on the reliability of structures, produced in 1984 by the International Organisation for Standardisation (I.S.O.) suggests that characteristic values could be used in soil calculations.

The Factor of safety approach has the further disadvantage that, once the design values have been selected, different but equally acceptable, design calculations can produce different values for the factor of safety (Burland et al, 1981; Symons, 1983).

Both of these disadvantages, the choice of the design parameter values and the choice of the design method, are considerably reduced, completely removed in many cases, if the safety of the proposed structure is determined from a reliability analysis.

1.3 Probability of failure

The factor of safety of a geotechnical structure is really a variable and its variability is due exclusively to the variability of the applied loads and the soil parameters involved. If failure is defined as the event of F achieving a value equal to or less than 1.0 then the probability of this event is equal to the probability of failure, P_f.

$P_f = P[\text{Failure}] = P[F \leqslant 1]$

Consider the resistance or strength of a structure, R and the applied loading, S, to which it will be subjected. The value of both R and S are not fixed but can assume any value within a range of values. The extent of these ranges will vary with the degree of reliability decided as acceptable for the design problem, (usually 95 per cent). R and S are therefore random variables with definitive, although possibly unknown, probability density functions (p.d.f's).

Figs. 1A and B show assumed p.d.f's for R and S and illustrate that failure will occur when $R < S$. If Fig. 1B is subtracted from Fig. 1A then the probability curve of $Z = R - S$, (strength minus load), is obtained, Fig. 1C.

The probability of failure, $P_f = P[(R - S) \leqslant 0] = P[Z \leqslant 0]$

It should be noted that the term, 'failure', is used here in its most general sense and implies the failure of the structure to satisfy some particular limit state criterion, which may or may not be actual structural failure.

In practical problems R and S will rarely consist of single variables but will be vectors made up from the set of the relevant variables, $X_1, X_2, X_3, \ldots\ldots X_n$.

Z is a function of R and S and can be expressed as $Z = g(R,S)$

Hence $Z = g(X_1, X_2, X_3, \ldots\ldots\ldots X_n) = g(X)$

ie $P_f = P[Z \leqslant 0] = P[g(X_1, X_2, X_3, \ldots\ldots X_n) \leqslant 0]$
$= P[g(X) \leqslant 0]$

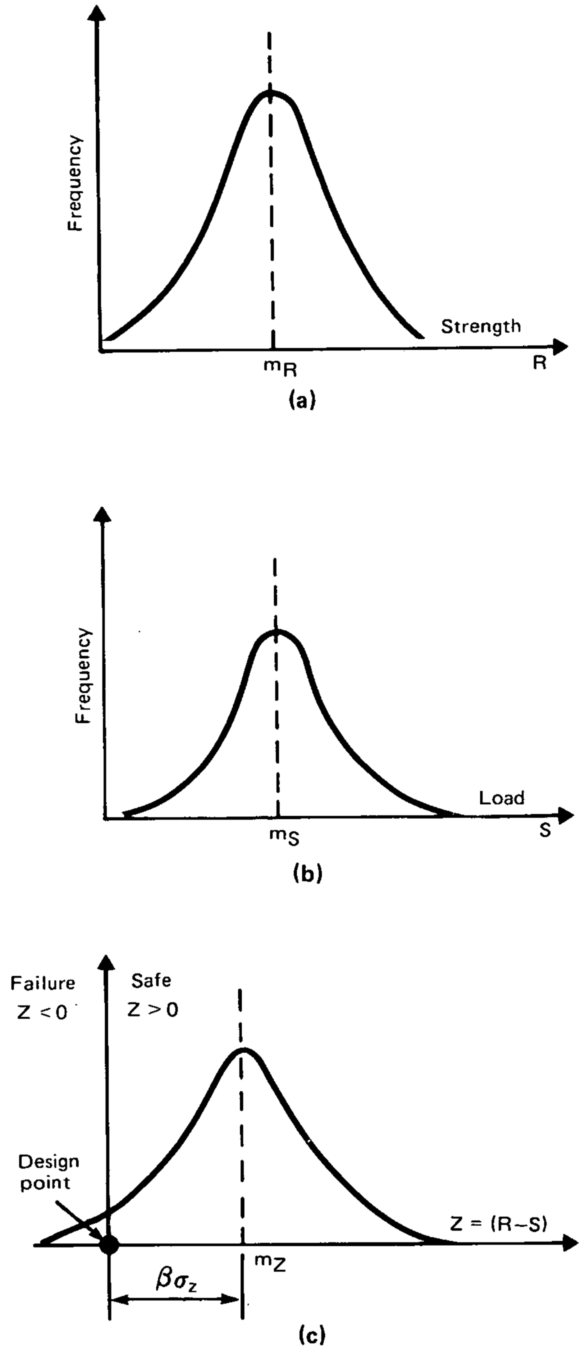

Fig. 1

1.4 The reliability index

Generally there is not sufficient information regarding the tails of the Z distribution and the criterion $P_f = P[Z \leqslant 0]$ is therefore replaced with one that involves the mean value and standard deviation of Z.

In Fig. 1C the distance from the mean of Z, m_z, to the failure boundary, ie the point at which $Z = 0$, can be expressed in terms of σ_z, the standard deviation of Z, and equals $\beta\sigma_z$. β is known as the reliability index and is a measure of the safety of the system.

Obviously $m_z - \beta\sigma_z = 0$ ie $\beta = m_z/\sigma_z$

Now $m_z = m_R - m_S$ Hence $\beta = \frac{m_R - m_S}{\sigma_z}$

The factor of safety, F, is equal to R/S, the values of the variables comprising R and S being selected by some arbitrary process, such as using the minimum, mean or characteristic values.

The expression for F is purely deterministic whereas the expression for β includes not only m_R and m_S but also σ_z, a measure of the uncertainty of both R and S. It can therefore be seen that β is a more meaningful measure of reliability than F.

In this report a method is described for the assessment of the probability of failure of earth retaining structures. The method is based on the second moment method of reliability analysis which has hitherto been applied mainly to structural design.

2 THE PROPOSED METHOD

A first order approximation of the second moment method of reliability analysis by which the probability of failure of a structure, P_f, can be determined, was proposed by Hasofer and Lind (1974) and extended by Rackwitz (1976). The method has been used very successfully in structural design work and it was considered that it should be capable of adaption to geotechnical problems. Its use for structural design has been well documented and the theory will not be gone into here.

Briefly the procedure is to obtain an expression for Z in terms of X_i, the basic variables representing the load, material properties, structural dimensions, etc. Hence $Z = g(X_i)$ is the limit state equation, ie the equation of the failure boundary of the problem to be analysed. Note that $Z = g(X_i)$ is usually written as $Z = g(X)$ when there is no risk of ambiguity in dropping the suffix i.

At a suitable point this expression is approximated to a linear first degree equation, by means of a Taylor's expansion in which 2nd order terms and above are ignored. The approximation is carried out at the 'design point', the point where there is the maximum probability of failure, (Fig. 1C).

Provided that the variables involved have probability distributions that are close to normal and provided that the linear approximation of the failure boundary is realistic then an exact value for P_f can be obtained from the expression:—

$$P_f = \Phi(-\beta)$$

where $\Phi(-\beta)$ is the general symbol for the value of the cumulative probability of Z from $-\infty$ to $-\beta$. This value can be obtained from tables or from a suitably programmed micro computer or calculator. It is the area under the standardised normal density function and is illustrated in Fig. 2.

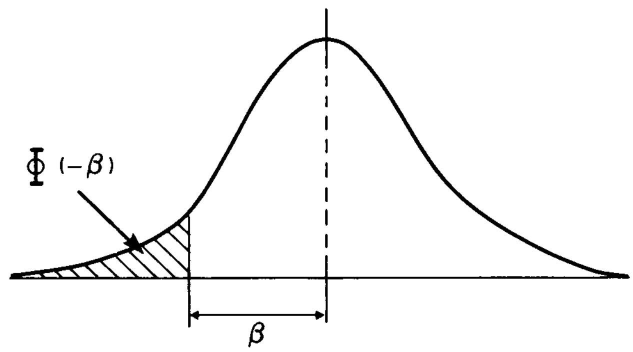

Fig. 2

If the variables depart from normal or if the linear approximation is poor then the P_f value obtained from the above formula is referred to as the 'notional, (or nominal), probability of failure'. Because of the inevitable lack of statistical data with geotechnical design the P_f values determined will generally be nominal.

For almost all engineering problems the linear approximation of the failure boundary will be adequate and it is worth remembering that if there are several variables, of roughly equal weight, the resulting Z function tends to be normal, even when the separate variables are not themselves normally distributed (Benjamin & Cornell 1970).

Nevertheless if it is known that some of the variables involved in the design are non-normal then the accuracy of the determined value of P_f is improved if this information is incorporated into the reliability analysis. This can be achieved by a method proposed by Fiessler and Rackwitz, (1976); details of the method will be presented in a subsequent publication.

2.1 Determination of the Reliability Index β

It has been pointed out previously that the expression representing the limit state boundary, $Z = g(X)$, is approximated at the design point by a linear equation. The reliability index, β, can be found by the fact that the minimum distance from the mean point, $(m_1, m_2, m_3, \ldots\ldots m_n)$ to the design point is equal to $\beta\sigma_z$.

A method for the determination of β has been proposed by the Construction Industry Research and Information Association (C.I.R.I.A.), Report No. 63 (1976), which

TABLE 1

1. Determine an expression for $Z = g(X)$.
2. From $g(X)$ evolve an expression for $h(y)$.
3. Determine expressions for all first derivatives of $h(y)$, h'_i.
4. Set $y_i = 0$ and $\beta = 0$.
5. Evaluate all h_i' values.
6. Evaluate $h(y)$.
7. Evaluate standard deviation of Z from $\sigma_Z = \sqrt{\Sigma(h'_i)^2}$
8. Evaluate new values for y_i from $y_i = -\frac{h'_i}{\sigma_z}\left[\beta + \frac{h(y)}{\sigma_z}\right]$
9. Evaluate $\beta = \sqrt{\Sigma y_i^2}$
10. Repeat steps 5 to 9 until values converge.

involves the direct application of the variables, X_1, X_2, X_3, ... but the method is time consuming.

A simpler approach is possible if the work is carried out in terms of reduced variables based on an algorithm proposed by Fiessler, (1980). The method gives a means by which β can be found directly from only one set of iterations.

If x_1 is the particular value of a variable x_1 with a mean of m_1 and a standard deviation of σ_1, then the corresponding reduced variable, y_l, is given by the expression: —

$$y_1 = \frac{x_1 - m_1}{\sigma_1}$$

A reduced variable is signified by a small lower case letter, usually z or y. It has a mean value of 0 and a standard deviation of 1.0 so that the origin of the axes representing this reduced space is also the mean point of the reduced variables.

The function $Z = g(X)$ can therefore be written as $Z = h(y)$ where $h(y) = h(y_1, y_2, y_3, \ldots\ldots\ldots y_n)$ and the reliability index is simply the minimum distance from the origin to $h(y)$ and can be obtained from the expression: —

$$\beta = \sqrt{\sum_{i=1}^{n} y_i^2}$$

A suitable procedure to determine β is shown in Table 1 and worked examples are given in Section 4.

There are obviously occasions when a particular limit function has a resistance, R, far in excess of the disturbing load, S. In such a situation the probability of failure is extremely small and the value of β to make Z equal to zero is so large that at least some of the resistance variables achieve unrealistically low, possibly negative, values.

The value of P_f that is acceptable for structural design purposes depends upon both the economic and the social consequences of failure. Table 2 lists typical P_f values for various degrees of structural failure and is based upon values listed in a Building Research Establishment report on limit state design published in 1981.

As seen from the table the acceptable risk for a catastrophe such as a nuclear explosion, is about 10^{-9} but such a risk is outwith normal structural engineering considerations and the minimum P_f value to be considered in structural design is therefore about 10^{-7}.

A P_f value of 10^{-7} corresponds to a β value of 5.2. It appears sensible therefore, in order to avoid small or negative variables, to accept that the risk of failure is negligible when the value of Z is so large that β must exceed 5.2 for it to be reduced to zero.

The P_f values listed in Table 2 are the nominal risk values generally accepted in the United Kingdom for structural engineering.

On the other hand Meyerhof (1982) suggests the values shown in Table 3 as acceptable nominal P_f values for geotechnical engineering.

TABLE 2

Degree of damage	Nominal P_f value	Corresponding β value
Inconvenient eg Cracked paving slabs	1 in 10 ie 10^{-1}	1.28
Minor repairs necessary	1 in 1,000 ie 10^{-3}	3.10
Major repairs necessary	10^{-5}	4.30
Major damage, possible casualties	10^{-7}	5.20
Catastrophic	10^{-9}	6.00

TABLE 3

	Maximum P_f	Corresponding β value
Earthworks	10^{-2}	2.33
Earth retaining structures	10^{-3}	3.10
Offshore foundations	10^{-3}	3.10
Onshore foundations	10^{-4}	3.80

3 BASIC VARIABLES IN GEOTECHNICAL PROBLEMS

A large number of the variables involved in geotechnics are functions of ϕ, the angle of friction of the soil.

Examples: $K_a = \frac{1-\sin\phi}{1+\sin\phi}$; $\mu = \tan\phi$; etc.

The proposed method involves regarding these functions of ϕ as forming a set of independent variables, each with its own mean value (m) and its own standard deviation (s.d.). The assumption of independence of the basic variables is often unrealistic for a structural engineering problem but is considered to be appropriate for a geotechnical situation.

Soil disturbance occurs during construction and this disturbance will be greatest at the boundaries between the soil and the structure. It would be naive to assume that the angle of wall friction of a driven sheet pile wall is related to the angle of shearing resistance of the undisturbed soil a metre or so away from it. Similarly the soil adjacent to a cantilever retaining wall may have been subjected to large compactive stresses during construction so that the angle of wall friction is independent of the angle of shearing resistance of the backfill soil.

3.1 The standard deviation of Z

It can be shown, Benjamin & Cornell, (1970), that if $Z = g(X)$ where $X = (X_1, X_2, X_3, \ldots\ldots\ldots X_n)$

then $\sigma_Z = \sqrt{\sum_{i=1}^{n} (g_i' \sigma_x)^2}$

where $g'_i = \left.\frac{\partial Z}{\partial X}\right|_{m_i}$ the first derivative of Z with $X_i = m_i$ (ie evaluated with the mean values of all the X variables).

If X is a single variable then $\sigma_Z = \sqrt{\left(\left.\frac{\partial Z}{\partial X}\right|_{m_x} \cdot \sigma_x\right)^2}$

When reduced variables are used the expression for σ_Z becomes:—

$$\sigma_Z = \sqrt{\sum_{i=1}^{n} (h_i \sigma_{y_i})^2}$$

Now $\sigma_{y_i} = 1.0$ when y_i is a reduced variable and the expression simplifies to:—

$\sigma_Z = \sqrt{\Sigma (h'i)^2}$ (which is step 7 of Table 1)

3.2 Bearing capacity factors

Meyerhof's equations (1955) for the bearing capacity coefficients N_c and N_q are now generally used in geotechnics as they are recognised as probably being the most satisfactory.

$$N_c = (N_q - 1)\cot\phi \qquad N_q = \tan^2(45° + \tfrac{\phi}{2})\ \exp(\pi \tan\phi)$$

Unfortunately there is not the same agreement about the remaining factor, $N\gamma$ and in this report the equation proposed by Hansen, (1970) has been employed:

$N\gamma = 1.5(Nq - 1)\tan\phi$

ie $N\gamma = 1.5\tan\phi[\tan^2(45° + \frac{\phi}{2})\ \exp(\pi\tan\phi) - 1.5\tan\phi$

The mean value of $N\gamma$ can be found by inserting the mean values of ϕ into the above equation.

The standard deviation of $N\gamma$ can be found from the expression:—

$$\sigma_{N_\gamma} = \left.\frac{\partial N_\gamma}{\partial \phi}\right|_{m_\phi} \cdot \sigma_\phi$$

which involves the differentiation of the equation for N_γ.

Although tedious, the differentiation is relatively simple and leads to the expression:—

$$\frac{\partial N_\gamma}{\partial \phi} = 1.5\tan\phi[\frac{2\cos\phi}{(1-\sin\phi)^2} \exp(\pi\tan\phi) + \tan^2(45° + \tfrac{\phi}{2})\pi\sec^2\phi\ (\exp\pi\tan\phi)] + 1.5\sec^2\phi[\tan^2(45° + \tfrac{\phi}{2})\exp(\pi\tan\phi) - 1]$$

Values of $N\gamma$, N_q, N_c and their first derivatives for a range of ϕ values are given in appendices I to III.

3.3 Determination of s.d. values without differentiation

Some of the functions concerned with ϕ in geotechnics are fairly complicated and their differentiation can present problems.

One method of avoiding this difficulty is to determine the values of the function for ϕ values one standard deviation on either side of the mean value of ϕ. The standard deviation of the function is then approximately equal to half of the difference between the two values.

4 APPLICATION OF THE METHOD TO A SIMPLE RETAINING WALL

The proposed reliability analysis method will now be used to determine the probability of failure of the retaining wall illustrated in Fig. 3.

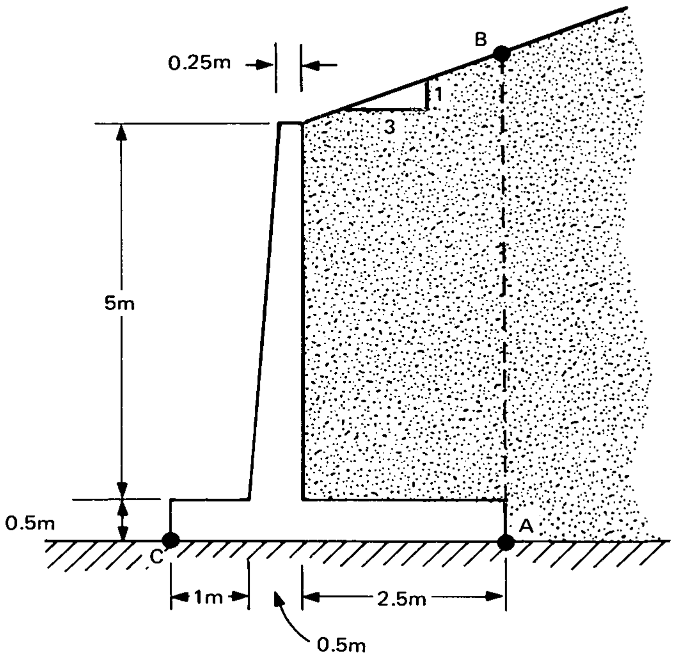

Fig. 3

The relevant properties of the two soils are assumed to be:—

Fill material—a granular soil with no cohesion.
Angle of friction, ϕ_1:—Mean = 33°; s.d. = 1.5°
Unit weight, γ_1:—Mean = 18kN/m^3; s.d. = 1kN/m^3
Foundation soil—a granular soil with no cohesion.
Angle of friction, ϕ_2:—Mean = 37°; s.d. = 2.0°
Unit weight, γ_2:—Mean = 19kN/m^3; s.d. = 1.5kN/m^3

The coefficient of friction, μ, between the wall base and the foundation soil can be taken as equal to $\tan\phi_2$. All variables may be assumed to have normal distributions and the dimensions of the structure and the unit weight of the concrete (24kN/m^3) taken as constant.

The coefficient of active earth pressure, K_a, can be taken as equal to the Coulomb value:—

$$K_a = \left(\frac{\operatorname{cosec}\psi \sin(\psi - \phi)}{\sqrt{\sin(\psi = \delta)} + \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - i)}{\sin(\psi - i)}}} \right)^2$$

where ψ = angle of back of wall to the horizontal, δ = angle of wall friction, i = angle of inclination of surface of retained soil to the horizontal and ϕ = angle of friction of the retained soil.

Determine the reliability index for each failure mode given that the probability of failure by rotational slip is less than 10^{-7} and may be considered negligible.

Solution

The investigation of the stability of a retaining wall involves the examination of four modes of failure:—

(i) Sliding
(ii) Overturning
(iii) Bearing capacity
(iv) Rotational slip

4.1 Sliding

The first step is to determine the limit state function for sliding.

Height of $AB = 5.5 + \frac{2.5}{3} = 6.333$m

Thrust from soil, $P_a = 0.5K_a\gamma_1 6.333^2 = 20.055\gamma_1 K_a$
$P_{aH} = P_a\cos\phi_1 (= S)$; $P_{aV} = P_a\sin\phi_1$

R = the resistance to sliding = $\mu \times$ Vertical reaction = μR_v
where R_v = Weight of wall + soil on heel + P_{aV}

$$= 24(5x.375 + 4x.5) + 13.543\gamma_1 + 20.055K_a\gamma_1\sin\phi_1$$
$$= 93 + 13.543\gamma_1 + 20.055K_a\gamma_1\sin\phi_1$$

$$Z = R - S$$
$$= (93 + 13.543\gamma_1 + 20.055K_a\gamma_1\sin\phi_1)\mu - 20.055K_a\gamma_1\cos\phi_1$$

The basic variables are therefore γ_1, K_a, $\cos\phi_1$, $\sin\phi_1$ and μ. If we designate them as X_1 to X_5 respectively then:—

$$Z = g(X) = (93 + 13.543X_1 + 20.055X_1.X_2.X_4)X_5 - 20.055X_1.X_2.X_3$$

The means and standard deviations of these variables found by the simplified method suggested in Section 3.3 are shown in Table 4.

TABLE 4

Variable		Mean(m)	Standard deviation (σ)
γ	X_1	18	1
K_a	X_2	.3546	.0236
$\cos\phi_1$	X_3	.8387	.0143
$\sin\phi_1$	X_4	.5446	.0220
μ	X_5	.7536	.0548

So far the limit state function, Z, has been expressed in the form Z = g(X). It is a straightforward matter to express Z in terms of reduced variables by substituting $(y_i\sigma + m_i)$ for X_i.

Hence:—Z = h(y)

$$= [93 + 13.543(y_1\sigma_1 + m_1) + 20.055(y_1\sigma_1 + m_1)(y_2\sigma_2 + m_2)(y_4\sigma_4 + m_4)](y_5\sigma_5 + m_5) - 20.055(y_1\sigma_1 + m_1)(y_2\sigma_2 + m_2)(y_3\sigma_3 + m_3)$$

The determination of the various first derivatives of h(y) is a straightforward procedure since it is a first degree equation. One example will be given.

$$h'_4 = 20.055\sigma_4(y_1\sigma_1 + m_1)(y_2\sigma_2 + m_2)(y_5\sigma_5 + m_5)$$

Once the expressions for the first derivatives have been obtained the iterative procedure for determining β can be carried out using the procedure described in Table 1.

The steps necessary for the first two iterations are as follows.

First iteration

Step 4 Putting $\beta=0$ and $y_1=y_2=y_3=y_4=y_5=0$

Step 5 $h'_4=20.055\times0.0220\times18\times0.3546\times0.7536=2.12$ using the mean and standard deviation values from Table 4.

Similarly $h'_1=7.16$, $h'_2=-3.65$, $h'_3=-1.83$ and $h'_5=22.27$

Step 6 $Z=[93+13.543(18)+20.055(18)(.3546)(.5446)](.7536)-20.055(18)(.3546)(.8387)=198.97$

Step 7 $\sigma_z=\sqrt{7.16^2+3.65^2+1.83^2+2.12^2+22.27^2}=23.84$

Step 8 $y_1=\dfrac{-7.16}{23.84}\left[0+\dfrac{198.7}{23.84}\right]=-2.51$

Similarly $y_2=1.28$; $y_3=0.64$; $y_4=-0.74$; $y_5=-7.80$

Step 9 $\beta=\sqrt{2.51^2+1.28^2+0.64^2+0.74^2+7.80^2}=8.35$

Second iteration

Step 5 $h'_1=-0.79$; $h'_2=-4.95$; $h'_3=-1.71$; $h'_4=0.86$; $h'_5=20.06$

Step 6 $Z=18.12$

Step 7 $\sigma_z=20.76$

Step 8 $y_1=-2.51$; $y_2=1.28$; $y_3=0.64$; $y_4=-0.74$; $y_5=-8.91$

Step 9 $\beta=8.35$

The whole procedure is best computerised and the full iterative procedure gives:—

	y_1	y_2	y_3	y_4	y_5	β	h(y)
1	0	0	0	0	0	0	198.97
2	−2.51	1.28	0.64	−0.74	−7.80	8.35	18.12
3	0.35	2.20	0.76	−0.38	−8.91	9.22	−15.04
4	0.77	2.19	0.76	−0.31	−8.23	8.59	−0.05
5	0.53	2.14	0.77	−0.36	−8.26	8.59	−0.09
6	0.54	2.14	0.76	−0.35	−8.26	8.59	−0.00
7	0.54	2.14	0.76	−0.35	−8.26	8.59	−0.00

The Reliability index = 8.59 and, as this is greater than 5.2, the probability of failure can be considered as negligible.

4.2 Overturning

Taking moments about point C, the toe of the wall, establishes:—

Resistive moments (R)

M due to wall stem and slab = 154.75kNm

M due to soil on heel $=37.68\gamma_1$

M due to P_{av} $=80.22K_a\gamma_1\sin\phi_1$

Disturbing moment (S)

$S=42.34K_a\gamma_1\cos\phi_1$ (assuming that PaH acts at 0.333AB above base of slab)

$Z=R-S$

$=154.75+37.68\gamma_1+80.22K_a\gamma_1-42.34K_a\gamma_1\cos\phi_1$

$h(y)=154.75+37.68(y_1\sigma_1+m_1)+80.22(y_1\sigma_1+m_1)(y_2\sigma_2+m_2)(y_4\sigma_4+m_4)-42.34(y_1\sigma_1+m_1)(y_2\sigma_2+m_2)(y_3\sigma_3+m_3)$

where $X_1=\gamma_1$; $X_2=K_a$; $X_3=\cos\phi_1$; $X_4=\sin\phi_1$

The iterative procedure gives:—

Iteration	y_1	y_2	y_3	y_4	β	h(y)
1	0	0	0	0	0	884.99
2	−19.95	−1.71	1.90	−5.54	20.86	82.95
3	−23.10	0.08	−0.23	0.68	23.11	−54.60
4	−21.71	0.61	−0.57	1.68	21.79	−0.61
5	−21.73	0.52	−0.43	1.25	21.77	−0.21
6	−21.72	0.49	−0.43	1.26	21.77	−0.00
7	−21.72	0.49	−0.43	1.26	21.77	−0.00

The risk of failure by overturning is negligible as $\beta>5.2$.

4.3 Bearing Capacity

For a surface strip footing the ultimate bearing pressure, q_u, is given by the expression:—

$$q_u=0.5B\gamma N_\gamma i_\gamma$$

where B = width of foundation
γ = unit weight of supporting soil
N_γ = bearing capacity coefficient
i_γ = inclined load factor

The ultimate vertical load, Q_u, that can act on the foundation is found from the expression:—

$$Qu=B'q_u$$

where $B'=(B-2e)$
and e = eccentricity of R_v (the vertical reaction)

Hence $R=Q_u=0.5(B-2e)B\gamma_2N_\gamma i_\gamma=2(4-2e)\gamma_2N_\gamma i_\gamma$

and $S=R_v=93+13.543\gamma_1+20.055K_a\gamma_1\sin\phi_1$

$\Rightarrow=2(4-2e)\gamma_2N_\gamma i_\gamma-93-13.543\gamma_1-20.055K_a\gamma_1\sin\phi_1$

The basic variables are γ_1, γ_2, K_a, $\sin\phi_1$, N_γ, i_γ and e. The mean and standard deviations of the first four have already been determined.

4.3.1 The bearing capacity coefficient N_γ

The treatment of this factor has been discussed in Section 3.2.

Assuming that the mean value of N_γ corresponds to $\phi=37°$ then the mean value of N_γ is 47.38 and the value of $\partial N_\gamma/\partial\phi$ is 459.03 (From Appendix I). Hence σ_{N_γ} is $459.03\times\dfrac{2\pi}{180}=16.02$.

4.3.2. The inclined load factor i_γ

Various expressions have been proposed for i_γ and the one used herein is that suggested by Sokolovski, (1960):—

$$i_\gamma = \left(1 - \frac{P_{aH}}{R_v}\right)^3$$

Now P_{aH} = Horizontal thrust $= 20.055K_a\gamma_1\cos\phi_1$
Rv $= 93 + 13.542\gamma_1 + 20.055K_a\gamma_1\sin\phi_1$

$$\Rightarrow i_\gamma = \left(1 - \frac{20.055K_a\gamma_1\cos\phi_1}{93 + 13.543\gamma_1 + 20.055K_a\gamma_1\sin\phi_1}\right)^3$$

Mean value for i_γ (for $\phi_1 = 33°$ and $\gamma_1 = 18kN/m^3$) $= 0.4378$

It can be shown that i_γ is, for all practical purposes, only sensitive to changes in the value of ϕ. Hence, assuming that γ_1 is constant at $18kN/m^3$ then σ_{i_γ} is found (by the approximate method of 3.3) to be $= 0.0338$.

4.3.3 The eccentricity e

$$e = \left|\frac{M}{R_v} - \frac{B}{2}\right| = \left|\frac{M}{R_v} - 2\right|$$

Taking moments about A, the heel of the wall:—

$$e = \left|\frac{217.25 + 16.49\gamma_1 + 42.306K_a\gamma_1\cos\phi_1}{93 + 13.543\gamma_1 + 20.055K_a\gamma_1\sin\phi_1} - 2\right|$$

Mean value of e (when $\phi_1 = 33°$ and $\gamma_1 = 18kN/m^3$) $= 0.1777$

By the approximate method, assuming γ_1 constant at $18kN/m^3$, the standard deviation of e works out at 0.0384.

Designating the variables as X_1 to X_7:—

Variable		Mean	s.d
γ_1	$= X_1$	18	1
γ_2	$= X_2$	19	1.5
K_a	$= X_3$	.3546	.0236
$\sin\phi_1$	$= X_4$	.5446	.0220
N_γ	$= X_5$	47.38	16.02
i_γ	$= X_6$	.3986	.0338
e	$= X_7$	.1777	.0384

$$Z = g(X) = 2(4 - 2X_7)X_2.X_5.X_6 - 93 - 13.543X_1 - 20.055X_1.X_3.X_4$$

$$h(y) = 2[4 - 2(y_7\sigma_7 + m_7)](y_2\sigma_2 + m_2)(y_5\sigma_5 + m_5)(y_6\sigma_6 + m_6) - 93 - 13.543(y_1\sigma_1 + m_1) - 20.055(y_1\sigma_1 + m_1)(y_3\sigma_3 + m_3)(y_4\sigma_4 + m_4)$$

Iteration gives:—

Iteration	y_1	y_2	y_3	y_4	y_5	y_6	y_7	β	h(y)
1	0	0	0	0	0	0	0	0	2,209
2	0.04	−0.52	0.01	0.01	−2.23	−0.56	0.07	2.36	177
3	0.06	−0.15	0.01	0.01	−2.57	−0.17	0.02	2.58	−73.8
4	0.05	−0.08	0.01	0.01	−2.49	−0.08	0.01	2.50	−1.04
5	0.05	−0.09	0.01	0.01	−2.49	−0.10	0.01	2.49	−0.10
6	0.05	−0.09	0.01	0.01	−2.49	−0.10	0.01	2.49	−0.00

The reliability index is 2.49 and the nominal $P_f = 0.0064$

4.4 Comparison between F and P_f values

The factors of safety, based on various values of the basic variables, are:—

	Minimum	Mean	Characteristic
Sliding	1.87	2.85	2.17
Overturning	3.99	4.91	4.27
Bearing capacity	1.77	6.44	2.58

The probabilities of failure are:—

Sliding	—negligible ($\beta = 8.59$)
Overturning	—negligible ($\beta = 21.77$)
Bearing capacity	—6.4×10^{-3} ($\beta = 2.49$)

It is interesting to note that, whilst the factors of safety based on the mean values of the variables gives a high factor of safety against bearing capacity failure, the probability of bearing capacity failure is unacceptable according to Table 3.

A check on the method, based on Monte Carlo simulations with not less than 10,000 iterations, gave the following values for β:—

Sliding	8.91
Overturning	23.02
Bearing capacity	2.58

5 CONCLUSION

5.1 The Factor of Safety

The traditional factor of safety method is satisfactory for civil engineering design work when the calculated value of F can be compared with a predetermined allowable value, based on previous experience with similar structures and site conditions.

This statement applies when, either the variabilities of the load and resistance parameters are insignificant, or when the value of F is large enough to prevent failure due to parameter variability.

The technique has the following disadvantages:—

1. In order to obtain a meaningful measure of the safety of the structure the value obtained for F must be compared with a predetermined acceptable value, obtained in the light of previous experience with similar structures.
   This may prove difficult for a 'one off' structure.

2. The actual values chosen for the design parameters are arbitrarily selected from test results which may have wide ranges of values.
3. The technique is purely deterministic with the chosen design values being regarded as constants. There is no way to allow for any variation in the values of the design parameters.
4. Different, but equally acceptable, design methods can lead to the situation of different F values for the same problem.

5.2 The Reliability Index

There is a growing body of opinion that the use of the reliability index in civil engineering design would be a more natural approach and would remove some of the disadvantages associated with the factor of safety. This report describes a method of reliability analysis for earth retaining structures the calculations for which can be carried out on a programmable calculator such as the HP41 or TI59 or similar. The effect of non-normality of the variables on the accuracy of the predictions obtained will be considered in a subsequent publication.

However there are indications that the proposed method is robust enough for departures from normality to be safely ignored in many instances. There appears to be no reason why the method should not be successfully applied to other forms of geotechnical structures and an investigation along these lines is proceeding.

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APPENDIX I

NUMERICAL VALUES OF N_γ AND ITS FIRST DERIVATIVE

ϕ (Degrees)	N_γ	$\partial N_\gamma/\partial\phi$
0.00	0.00	0.00
1.00	0.00	0.29
2.00	0.01	0.62
3.00	0.02	1.00
4.00	0.05	1.43
5.00	0.07	1.92
6.00	0.11	2.49
7.00	0.16	3.14
8.00	0.22	3.88
9.00	0.30	4.74
10.00	0.39	5.72
11.00	0.50	6.85
12.00	0.63	8.15
13.00	0.78	9.64
14.00	0.97	11.36
15.00	1.18	13.34
16.00	1.43	15.63
17.00	1.73	18.28
18.00	2.08	21.35
19.00	2.48	24.91
20.00	2.95	29.04
21.00	3.50	33.85
22.00	4.13	39.45
23.00	4.88	45.99
24.00	5.75	53.65
25.00	6.76	62.63
26.00	7.94	73.18
27.00	9.32	85.61
28.00	10.94	100.30
29.00	12.84	117.70
30.00	15.07	138.36
31.00	17.69	162.97
32.00	20.79	192.38
33.00	24.44	227.65
34.00	28.77	270.07
35.00	33.92	321.31
36.00	40.05	383.43
37.00	47.38	459.03
38.00	56.17	551.46
39.00	66.76	664.98
40.00	79.54	805.05
41.00	95.05	978.78
42.00	113.96	1,195.41
43.00	137.10	1,467.08
44.00	165.58	1,809.82
45.00	200.81	2,245.00
46.00	244.65	2,801.29
47.00	299.52	3,517.53
48.00	368.67	4,446.79
49.00	456.40	5,662.28
50.00	568.57	7,266.03

APPENDIX II

NUMERICAL VALUES OF N_q AND ITS FIRST DERIVATIVE

ϕ (Degrees)	N_q	$\partial N_q/\partial\phi$
0.00	1.00	5.14
1.00	1.09	5.63
2.00	1.20	6.16
3.00	1.31	6.75
4.00	1.43	7.39
5.00	1.57	8.11
6.00	1.72	8.90
7.00	1.88	9.78
8.00	2.06	10.75
9.00	2.25	11.83
10.00	2.47	13.02
11.00	2.71	14.36
12.00	2.97	15.84
13.00	3.26	17.50
14.00	3.59	19.36
15.00	3.94	21.43
16.00	4.34	23.76
17.00	4.77	26.37
18.00	5.26	29.32
19.00	5.80	32.64
20.00	6.40	36.39
21.00	7.07	40.63
22.00	7.82	45.45
23.00	8.66	50.93
24.00	9.60	57.17
25.00	10.66	64.31
26.00	11.85	72.48
27.00	13.20	81.86
28.00	14.72	92.66
29.00	16.44	105.13
30.00	18.40	119.57
31.00	20.63	136.35
32.00	23.18	155.90
33.00	26.09	178.76
34.00	29.44	205.59
35.00	33.30	237.18
36.00	37.75	274.54
37.00	42.92	318.89
38.00	48.93	371.76
39.00	55.96	435.08
40.00	64.20	511.27
41.00	73.90	603.41
42.00	85.37	715.42
43.00	99.01	852.33
44.00	115.13	1,020.66
45.00	134.87	1,228.92
46.00	158.50	1,488.25
47.00	187.21	1,813.45
48.00	222.30	2,224.23
49.00	265.50	2,747.24
50.00	319.06	3,418.69

APPENDIX III

NUMERICAL VALUES OF N_c AND ITS FIRST DERIVATIVE

ϕ (Degrees)	N_c	$\partial N_c/\partial \phi$
0.00	5.14	12.80
1.00	5.38	14.03
2.00	5.63	14.90
3.00	5.90	15.84
4.00	6.19	16.86
5.00	6.49	17.96
6.00	6.81	19.16
7.00	7.16	20.46
8.00	7.53	21.87
9.00	7.92	23.40
10.00	8.34	25.07
11.00	8.80	26.89
12.00	9.28	28.88
13.00	9.81	31.06
14.00	10.37	33.45
15.00	10.98	36.07
16.00	11.63	38.96
17.00	12.34	42.14
18.00	13.10	45.64
19.00	13.93	49.52
20.00	14.83	53.82
21.00	15.81	58.59
22.00	16.88	63.89
23.00	18.05	69.80
24.00	19.32	76.41
25.00	20.72	83.81
26.00	22.25	92.12
27.00	23.94	101.47
28.00	25.80	112.02
29.00	27.86	123.96
30.00	30.14	137.50
31.00	32.67	152.92
32.00	35.49	170.52
33.00	38.64	190.68
34.00	42.16	213.85
35.00	46.12	240.56
36.00	50.59	271.49
37.00	55.63	307.43
38.00	61.35	349.37
39.00	67.87	398.51
40.00	75.31	456.36
41.00	83.86	524.78
42.00	93.71	606.11
43.00	105.11	703.28
44.00	118.37	820.04
45.00	133.87	961.17
46.00	152.10	1,132.81
47.00	173.64	1,342.94
48.00	199.26	1,602.00
49.00	229.92	1,923.77
50.00	266.88	2,326.62

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