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A SIMPLIFIED METHOD TO CALCULATE DYNAMIC BEHAVIOUR OF FOOTBRIDGES

by

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CONTENTS

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ABSTRACT

A simple computer program to calculate the dynamic behaviour of footbridges is described. In accordance with recent design rules the fundamental natural frequency of the bridge superstructure and the acceleration response due to the passage of a pedestrian can be calculated. The accuracy of the program has been assessed by extensive comparisons with a much larger 'beam-element' program and with experimental measurements. Guidelines are given defining the range of applicability of the program and their use is demonstrated by reference to eleven existing bridges considered in earlier work. The fundamental natural frequency of many types of highway bridges can also be calculated.

1. INTRODUCTION

Design rules issued recently by the Department of Transport specify that the dynamic behaviour of footbridges due to pedestrian loading be checked[1]. The natural frequency of the fundamental mode of the structure must be calculated, and, when this is less than 5Hz, the acceleration response of the superstructure due to a pedestrian crossing the main span must be determined. For bridges of up to four spans having a superstructure of uniform cross section the fundamental natural frequencies can be readily found using, for example, the Tables presented by Gorman[2]. The maximum acceleration response of single, two-span, and symmetric three-span bridges having a uniform cross section can be calculated using an approximate formula given in the design rules,

ie $a = 4\pi^2 f^2 y_s K\psi$ . . . . (1)

where a = maximum acceleration response in metres/sec^2

f = fundamental natural frequency in Hz

y_s = static deflection, in metres, at the mid-point of the main span for a vertical concentrated load of 0.7 kN applied at that point

K = configuration factor depending on the number of spans and the span ratios

ψ = dynamic response factor depending on the main span length and the logarithmic decrement damping (δ).

Values of K and ψ are given in the rules. Also, in the absence of more precise information, it is recommended that δ values should be 0.03 for steel bridges, 0.04 for composite steel/concrete, and 0.05 for prestressed and reinforced concrete. Where the approximate formula for calculating acceleration is inapplicable the calculation should be based on assuming that the dynamic loading applied by a pedestrian can be simulated by a pulsating load F, moving across the main span at a constant speed v as follows:

$F = 180 \sin 2\pi ft$ (Newtons) . . . . (2)

where t = time in seconds

$v = 0.9f$ (metres/sec).

If the frequency, f, is greater than 4Hz, the calculated maximum acceleration may be reduced by an amount varying linearly from zero at 4Hz to 70 per cent reduction at 5Hz.

In order to be able to calculate natural frequencies and acceleration response when the methods referred to above are inappropriate, a computer program has been developed at the Laboratory specifically for the analysis of common types of bridges. The program is referred to as FREMOD (FREquency and MODe shape). It is based on representing a bridge as a connected set of beam elements, and is applicable to superstructures of varying cross section and to cantilever-and-suspended-span configurations. The main parameters calculated are natural frequencies, mode shapes, and the acceleration response due to the simulated pedestrian. Most of the development of the program was carried out by Traill-Nash[4]. It has subsequently been shown by Wills[5] and Tilly[6] that calculated natural frequencies and mode shapes for eleven road and footbridges are in close agreement with measured values.

Although FREMOD has been shown to give good agreement with measured values, it has the disadvantage that it requires a considerable amount of computer storage space. Simpler methods of calculation have therefore been investigated by the author. In earlier work[5] it was shown that, using published data for beams of uniform cross section, satisfactory estimation could be made of the natural frequencies of bridges having superstructure cross sections which varied significantly. A computer program based on formulae for uniform beams has been prepared therefore, and augmented with the same procedures used in FREMOD for calculating the acceleration response due to a pedestrian crossing the main span of the superstructure. The program is referred to by the name BRGVBN (BRidGe ViBratioN). Only about one quarter of the computer storage space needed by FREMOD is required.

The main purpose of the present Report is to give details of BRGVBN and to define its range of usefulness. It is assumed throughout that FREMOD gives reliable estimates of the natural frequency and acceleration response, so that the performance of BRGVBN is judged mainly by using FREMOD as the reference standard. An extensive parameter study comparing results from the two programs is described. The use of BRGVBN for eleven existing bridges is also examined.

2. THEORY

Program BRGVBN is restricted to consideration of the fundamental mode of vibration because trial calculations showed that inclusion of the influence of higher modes has a negligible effect on the dynamic behaviour to be checked. The theoretical basis of the program, outlined below, is an extension and adaptation of that presented previously in reference 5. Only the vertical vibration of straight horizontal bridge superstructures simply supported at their ends and any intermediate support points is considered. The superstructure is assumed to act as a continuous beam. Hence, span/width ratios must be greater than, say, 3:1, the minimum considered in reference 5.

Consider a beam of uniform section along its length and supported in a horizontal plane.

Let L = length of beam
EI = flexural rigidity
ρA = mass per unit length
x = distance along beam measured from one end
w = vertical displacement, assumed small in relation to length of beam
t = time.

Assume that the effects of (i) rotary inertia about an axis perpendicular to the beam, and (ii) deformation due to shearing of one cross section relative to an adjacent one can be neglected. It can then be shown[2,7], for vertical vibration of the beam, that the equation of motion for an element of length δx is

$$EI \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = 0 \qquad \text{. . . . . (3)}$$

Solving equation (1) gives

$$f_i = \frac{\beta_i^{\,2}}{2\pi L^2} \sqrt{\frac{EI}{\rho A}} \qquad \text{. . . . . (4)}$$

where f = natural frequency
i refers to the ith mode
and β is a parameter dependent on the span layout, span lengths, support conditions, and the mode number being considered.

Formulae are given by Gorman[2] for calculating the natural frequency and mode shape for any mode. The formulae are for beams of uniform section properties throughout their length, but they can be readily extended to uniform but different section properties for each span[8]. The generalised mass and generalised stiffness for each mode can also be calculated[9]. Within BRGVBN the natural frequency, mode shape and generalised mass and stiffness are calculated for the fundamental mode, and the response of the superstructure due to the passage of a pedestrian can then be determined. The forced response of a single degree of freedom system having viscous damping is considered. The pulsating load specified in the DTp design rules is assumed to move across only the span having the maximum amplitude of vibration in the fundamental mode.

For continuous beams having non-uniform section properties over one or more spans the procedure outlined above for calculating natural frequency is not strictly applicable. Nevertheless, as noted earlier, it has been found to give good estimates of the fundamental natural frequency even for quite large variations in section properties. Further studies have shown however that calculated values of natural frequency generally agree more closely with those found using FREMOD, if BRGVBN is based on the following procedure:

a) The weighted mean values of the section properties for each span are used to calculate mode shape and a first estimate of the natural frequency.
b) The mode shape from (a) is used with the actual distribution of section properties to calculate generalised mass and generalised stiffness.
c) The generalised mass and stiffness is used to calculate a second estimate of the natural frequency.
d) The minimum of the two estimates of frequency is selected as the best estimate of the fundamental natural frequency.

The minimum frequency, the generalised mass and the mode shape, and the damping ratio are then used to calculate the acceleration response.

3. COMPUTER PROGRAM

BRGVBN has been written in the Fortran IV language for running on the ICL System 4-70 Computer at the Laboratory. It consists of 500 cards and occupies 50k bytes of computer store. The running time for a typical problem is about thirty seconds.

3.1 *Input data*

The principal parameters to be input are the number of spans, the span length, details of the distribution of EI and ρA, the steplength and dynamic load of the simulated pedestrian, and the logarithmic decrement damping of the structure. The distributions of EI and ρA are input by considering the superstructure as a set of connected elements as shown in Figure 1. The data for each element consist of the element length and the values of EI and ρA at each end of the element; thus, elements having tapering section properties can be specified. Typical input data, for 1, 3 and 5-span examples, are shown in Figure 2.

Assumptions concerning the calculation of EI, when preparing the input data, are discussed in reference 5, and the methods described in Appendix C of reference 1 should be used (basing the calculation of second moment of area of any concrete members on the gross section ignoring the presence of reinforcement).

3.2 *Output*

Computer results for the data given in Figure 2 are shown in Figure 3. The results are self explanatory.

4. INVESTIGATION OF RANGE OF APPLICABILITY

Case studies of eleven existing bridges are presented, followed by details of a study undertaken to quantify the influence of the main parameters affecting the accuracy of BRGVBN. The accuracy is expressed in the form of three error terms:

(a) frequency error $\dfrac{\delta f}{f_{\text{FREMOD}}}$,

where $\delta f = f_{\text{BRGVBN}} - f_{\text{FREMOD}}$

(b) acceleration error $\dfrac{\delta a}{a_{\text{FREMOD}}}$,

where $\delta a = a_{\text{BRGVBN}} - a_{\text{FREMOD}}$

(c) comparative acceleration error $\dfrac{a'_{\text{BRGVBN}} - \frac{1}{2}\sqrt{f_{\text{BRGVBN}},}}{\frac{1}{2}\sqrt{f_{\text{BRGVBN}}}}$

where $a'_{\text{BRGVBN}} = a_{\text{BRGVBN}} \times \left(\frac{1}{2}\sqrt{f_{\text{FREMOD}}} \Big/ a_{\text{FREMOD}}\right)$

$\frac{1}{2}\sqrt{f}$ = maximum acceleration allowable[1], metres/sec^2.

The comparative acceleration error represents the inaccuracy in comparing the actual acceleration with the maximum acceleration allowable when the two are of similar magnitude.

Throughout the investigation of the range of applicability of BRGVBN the arbitrary assumption was made that the program is sufficiently accurate for practical purposes when the errors defined above are all within the range ± 10 per cent.

4.1 *Case studies of eleven bridges*

Eleven bridges for which calculated and measured frequencies are compared in references 5 and 6 were studied. Eight of the bridges are multispan and the remainder single span.

Considering firstly the eight multispan, these are listed in the left hand column of Table 1. All are three-span apart from Cripplegate footbridge (5-span) and Cleddau road bridge (7-span). Four have a suspended span in the main span. All except Cripplegate footbridge have non-uniform section properties; Figures 4(a)–(e) illustrate the variation of section properties of the five 3-span footbridges. Measured and calculated values of the fundamental natural frequency are also shown in Table 1, together with the calculated maximum acceleration for the simulated pedestrian crossing the main span of each footbridge. The accelerations were calculated using the recommended damping values referred to in Section 1. For the purposes of comparing BRGVBN with FREMOD it is assumed that the structures having a suspended span act in a continuous manner. Table 1 shows that the frequencies and accelerations calculated by BRGVBN are in good agreement with the corresponding FREMOD values even though some of the bridges have large variations in the section properties, (Figures 4(b)–(e)). All the frequency differences except one are within the range –8 per cent to +3 per cent, and all the acceleration differences are within –7 per cent to 0 per cent. The largest frequency difference (+ 17 per cent) is for Cottesmore footbridge which has the greatest variation of section properties. Table 1 also gives for the four suspended span structures the FREMOD calculated values of frequency and acceleration which include allowance for the influence of the suspended span. The corresponding BRGVBN values are in close agreement apart from the frequency for Cottesmore footbridge. The natural frequencies found by BRGVBN are also in good agreement with the measured values; for three bridges, including Cottesmore, BRGVBN gives a better estimate of the measured value than FREMOD, for a further three cases the two programs are virtually identical, and in two instances BRGVBN is worse than FREMOD.

Finally, considering the three single bridges, these have uniform or near uniform section properties. The bridges are listed in Table 2 together with measured and calculated values of natural frequency, and, for the one footbridge, calculated values of acceleration response. The BRGVBN and FREMOD values agree very closely.

TABLE 1

Multi-span bridges.
Natural frequencies and accelerations for fundamental mode

Bridge	Measured frequency (Hz)	Calculated frequency and acceleration and comparison with FREMOD values			
		Frequency (Hz)		Maximum acceleration (metres/sec^2)	
		FREMOD	BRGVBN	FREMOD	BRGVBN
FOOTBRIDGES:					
AYR	3.11	3.30	3.29 (–0.3%)	0.568	0.568 (0%)
WETHERBY					
a) With suspended span	2.62	2.62	–	0.877	–
b) Assuming acts as a continuous structure	–	2.84	2.61 (–8%)	0.833	0.812 (–3%)
BURY NEW ROAD	2.21	2.27	2.34 (+3%)	0.257	0.255 (–1%)
BRACKENDALE ROAD					
a) With suspended span	2.82	3.04	–	0.204	–
b) Assuming acts as a continuous structure	–	3.05	2.92 (–4%)	0.208	0.193 (–7%)
COTTESMORE					
a) With suspended span	1.89	1.69	–	0.219	–
b) Assuming acts as a continuous structure	–	1.71	2.00 (+17%)	0.231	0.214 (–7%)
CRIPPLEGATE	2.7	2.87	2.87 (0%)	0.187	0.187 (0%)
ROAD BRIDGES:					
WINDMILL ROAD	2.13	1.95	1.98 (+2%)	–	–
CLEDDAU MILFORD HAVEN					
a) With suspended span	0.530	0.522	–	–	–
b) Assuming acts as a continuous structure	–	0.536	0.514 (–4%)	–	–

TABLE 2

Single span bridges.
Natural frequencies and accelerations for fundamental mode

Bridge	Measured frequency (Hz)	Calculated frequency and acceleration and comparison with FREMOD values			
		Frequency (Hz)		Maximum acceleration (metres/sec^2)	
		FREMOD	BRGVBN	FREMOD	BRGVBN
EARLEY STATION footbridge (Span C)	3.12	3.18	3.18 (0%)	0.846	0.846 (0%)
RASHMIRE WOOD farm access bridge	2.5	2.44	2.40 (–2%)	–	–
PIRTON LANE road bridge	3.40	2.90	2.89 (–0.3%)	–	–

4.2 *Parameter study*

It is shown in Section 2 that the parameters influencing the fundamental natural frequency and acceleration response of any bridge that can be considered similar to a simply supported continuous beam are:

(a) The overall length of the superstructure and the ratios of individual span lengths.

(b) The flexural rigidity (EI) and its variation along the length of the superstructure.

(c) The mass per unit length (ρA) and its variation along the length of the superstructure.

The acceleration response is also influenced by the damping of the superstructure and the dynamic loading applied.

A systematic study was undertaken to examine how the above parameters influenced the accuracy of Program BRGVBN. Up to five spans were considered as shown in Figure 5. Main span lengths ranged from 20 to 50m. As shown in Figure 5 the number of possible span combinations was restricted, particularly for the four and five-span cases.

An indication of the range of variation of EI and ρA occurring in practice is given by Figures 4(a)–(e). Using these as a guide and by undertaking trial calculations, it was found appropriate to base the parameter study mainly on using the relative variations illustrated in Figure 6. These were considered to be reasonably representative of variations occurring in practice. Various ratios of EI_{max}/EI_{min} and $\rho A_{max}/\rho A_{min}$, including unity, were considered. Non-symmetric variations of EI and ρA were also studied.

The magnitudes of EI and ρA were chosen so that (i) the calculated fundamental natural frequency was in the range 1.5 to 4Hz and (ii) the maximum acceleration due to the simulated pedestrian was of the same order of magnitude as $\frac{1}{2}\sqrt{f}$.

Logarithmic decrement damping values of 0.03 and 0.05 were used, but no variation of the dynamic loading was examined because the value is defined in reference 1.

4.2.1 Conclusions from parameter study : The conclusions from the parameter study are summarised in Table 3. Although only a restricted number of span arrangements for up to five spans were examined, it is considered that the results are generally applicable to any number of spans and span arrangements.

TABLE 3

Conclusions from parameter study

Condition	Comparison of calculated frequencies and acceleration
1. Uniform EI and ρA over whole bridge	BRGVBN and FREMOD frequency and acceleration values agree exactly, and also with formula of reference 1.
2. Uniform EI and ρA over each span	BRGVBN and FREMOD agree exactly.
3. Non uniform EI and ρA over one or more spans Influence of:	
a) Span ratios and variation of EI	These are main parameters influencing the errors. Typical errors are illustrated in Figure 7. In general, for any particular span ratio, the errors increased as the variation of EI was increased. The comparative error was usually less than the acceleration error. Recommendations concerning the maximum allowable variation of EI are given later.
b) Variation of ρA	Some effect on the errors, but small compared with influence of variation of EI – can be ignored.
c) Magnitude of EI and ρA	No effect on errors.
d) Main span length e) Logarithmic damping	No effect on frequency error. Little effect on acceleration and comparative acceleration errors – can be ignored.

4.2.2 Guidelines defining range of applicability of BRGVBN: The main conclusions from the parameter study are that (i) for uniform EI and ρA over each span, BRGVBN agrees exactly with FREMOD, and (ii) for non-uniform conditions, the errors are largely dependent on the span ratios and the variation of EI. Thus, guidelines can be readily drawn up to show the range of applicability of BRGVBN. These are portrayed in the form of a flow diagram in Figure 8. In practice, many bridges satisfy the conditions detailed in the flow diagram so that BRGVBN can be used, providing errors of up to 10 per cent can be tolerated.

The limits on the errors quoted in Figure 8 are based on the worst errors found, apart from a few unimportant cases. In many instances, for variations of EI greater than the limits given in Figure 8, BRGVBN gave satisfactory accuracy. However, as the conditions for this are difficult to define in general terms, no attempt has been made to do so.

The practical use of the guidelines of Figure 8 can be partly demonstrated by reference to the frequency and acceleration errors of the eleven existing bridges discussed in Section 4.1. Considering firstly the five multispan footbridges having non-uniform section properties, all have side span lengths less than 60 per cent of the main span length. Only three, Ayr, Wetherby and Brackendale Road, have variations of EI less than 5:1, and Table 1 confirms that, as predicted by Figure 8, the frequency and acceleration errors for these three bridges are within 10 per cent. The remaining two footbridges, Brackendale Road and Cottesmore, have large variations of EI, of 10:1 and 50:1 respectively (Figures 4(d) and (e)), but even so, both acceleration errors are acceptable and only the frequency error of Cottesmore exceeds 10 per cent.

With regard to the two multispan road bridges, Windmill Road and Cleddau, these have variations in EI of 2:1 and 2.5:1 respectively. Table 1 confirms that, as predicted by Figure 8, the frequency errors are well within ± 10 per cent.

The three single span bridges referred to briefly at the end of Section 4.1, and also the 7-span Cripplegate footbridge, have very little variation of EI. Thus, as indicated by Figure 8, BRGVBN is satisfactory for these bridges.

5. CONCLUDING COMMENTS

A small computer program, written in the Fortran IV language and referred to as BRGVBN, has been prepared for checking the dynamic behaviour of footbridges due to pedestrian loading. The natural frequency of the fundamental mode of the bridge superstructure and the acceleration response due to the passage of a pedestrian are calculated. By means of a parameter study to compare results from the program with those from a much larger 'beam-element' program, it has been demonstrated that the accuracy of BRGVBN depends mainly on the variation of the flexural rigidity of the bridge superstructure and on the ratio of the span lengths. On the arbitrary basis that errors of up to ± 10 per cent in the calculated values of the parameters of interest are acceptable, guidelines are presented detailing the maximum allowable variation of flexural rigidity. Of eleven bridges considered in earlier studies comparing measured values of natural frequency with those calculated using the beam element program, nine satisfy the guidelines and it is shown that for all of these BRGVBN is satisfactory. The correlation between BRGVBN and measured values was, on average, as good as that demonstrated in the earlier studies. BRGVBN should be of use for many common types of footbridge design, and it can also be used for calculating the fundamental natural frequency of highway bridges.

6. ACKNOWLEDGEMENTS

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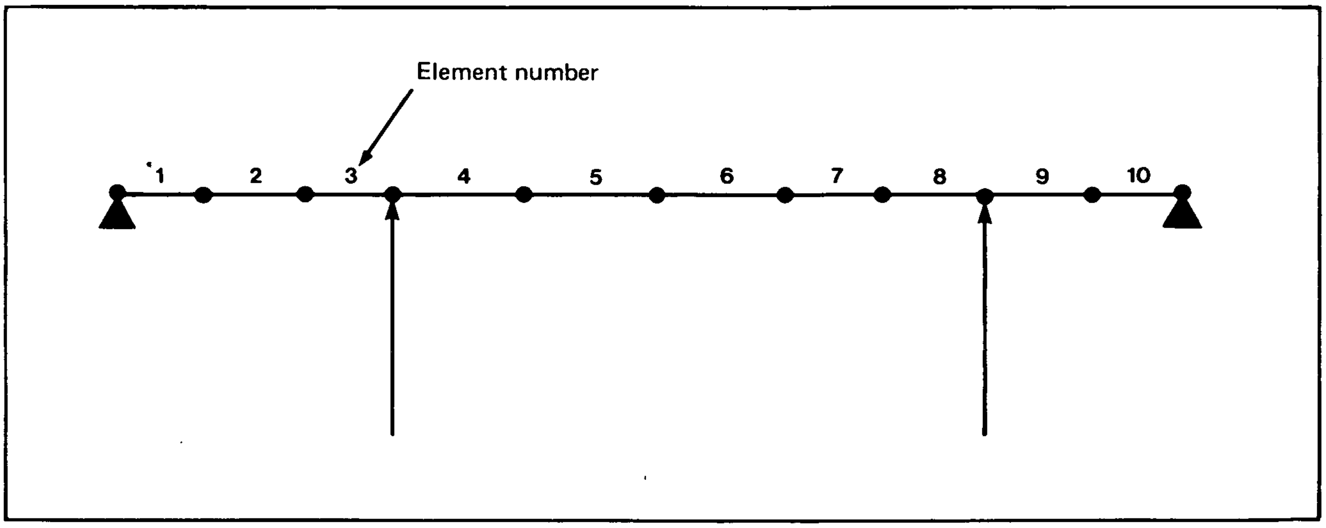

Fig. 1 EXAMPLE OF REPRESENTATION OF STRUCTURE AS CONNECTED SET OF BEAM ELEMENTS

```
1-SPAN   EIREL=.2 RAREL=.5  LMAIN=20
1     NO OF SPANS
20     SPAN LENGTH, METRES
4     NO OF ELEMENTS
1  5.0  432167  259300  1.3333  1.0
2  5.0  259300   86433  1.0  0.6667
3  5.0   86433  259300  0.6667  1.0
4  5.0  259300  432167  1.0  1.3333
0.9  0.18  0.03    STEPLENGTH=.9M, P=.18KN, LOGDEC(1)=.03
*
```

See explanation for 5-span bridge below

```
3-SPAN   EIREL=.2  RAREL=.5   LMAIN=20
3     NO OF SPANS
8  20  12     SPAN LENGTHS,METRES
8     8 ELMNTS
1  4.0   53550   53550  0.6667  1.0
2  4.0   53550  267750  1.0  1.3333
3  5.0  267750   53550  1.3333  1.0
4  5.0   53550   53550  1.0  0.6667
5  5.0   53550   53550  0.6667  1.0
6  5.0   53550  267750  1.0  1.3333
7  6.0  267750   53550  1.3333  1.0
8  6.0   53550   53550  1.0  0.6667
0.9  0.18  0.030   STPLNG,P,LOGDEC(1)
*
```

See explanation for 5-span bridge below

```
5-SPAN  EIREL=.2 RAREL=.5   LMAIN=20   .4:.6:1:.6:.4
5      NO OF SPANS
8  12  20  12  8     SPAN LENGTHS, METRES
16     NO OF ELEMENTS
1   4.0   54200    54200   0.6667  1.0
2   4.0   54200   271000   1.0  1.3333
3   3.0  271000    54200   1.3333  1.0
4   3.0   54200    54200   1.0  0.6667
5   3.0   54200    54200   0.6667  1.0
6   3.0   54200   271000   1.0  1.3333
7   5.0  271000    54200   1.3333  1.0
8   5.0   54200    54200   1.0  0.6667
9   5.0   54200    54200   0.6667  1.0
10   5.0   54200   271000   1.0  1.3333
11   3.0  271000    54200   1.3333  1.0
12   3.0   54200    54200   1.0  0.6667
13   3.0   54200    54200   0.6667  1.0
14   3.0   54200   271000   1.0  1.3333
15   4.0  271000    54200   1.3333  1.0
16   4.0   54200    54200   1.0  0.6667
0.9  0.18  0.03    STEPLENGTH=.9M, P=.18KN, LOGDEC(1)=.03
0.9  0.18  0.05    STEPLENGTH=.9M, P=.18KN, LOGDEC(1)=.05
*
```

Any title

SPAN 1 (elements 1–2)

SPAN 2 (elements 3–6)

SPAN 3 (elements 7–10)

SPAN 4 (elements 11–14)

SPAN 5 (elements 15–16)

Each line consists of
Element No., Element length,
EI at left hand end of element,
EI at right hand end of element,
pA at left hand end of element,
pA at right hand end of element

Element lengths must be such that positions of all supports coincide with element boundaries

Units of length (L), EI and pA must be such that $\frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}$ is $\frac{1}{\text{seconds}}$

Step length and dynamic load as specified in Reference 1

Any number of damping values may be specified. Leave out if only natural frequency required

Fig. 2 TYPICAL INPUT DATA

```
1-SPAN    EIREL=.2 RAREL=.5  LMAIN=20
NSPANS=  1
SPAN LENGTHS  20.000

  4 ELEMENTS
     ELLGTH        EILEFT       EIRIGHT       RALEFT       RARIGHT
  1   5.000        0.4322E 06   0.2593E 06    1.333         1.000
  2   5.000        0.2593E 06   0.8643E 05    1.000        0.6667
  3   5.000        0.8643E 05   0.2593E 06   0.6667         1.000
  4   5.000        0.2593E 06   0.4322E 06    1.000         1.333

AVERAGE EI FOR EACH SPAN
  0.2593E 06
AVERAGE EI FOR BRIDGE= 0.2593E 06

AVERAGE RA FOR EACH SPAN
   1.000
AVERAGE RA FOR BRIDGE=  1.000

MODE 1: NATURAL FREQ =  1.837


MODE 1 SHAPE,  DSPLS AT 1/8 POINTS

SPAN 1       0.000      0.383      0.707      0.924      1.000      0.924      0.707      0.383      0.000



STEPLENGTH=  0.900       P=  0.1800      LOGDEC(1)=0.0300

MODE 1   F= 1.84  VELOCITY=  1.653
MAX DSPL= 0.5191E-02 AND MAX ACCR= 0.6915      OCCUR ON SPAN 1 AT X=  10.00      WHEN PEDESTRIAN IS AT DIST=  16.65
                                                        (DISTANCES MEASURED FROM LEFT HAND SUPPORT OF SPAN 1)

MAX ALLOWABLE ACCR = 0.5*SQRT(F) = 0.6777      METRES/SECSQD
*************************************************************************************************************************


3-SPAN    EIREL=.2  RAREL=.5   LMAIN=20
NSPANS=  3
SPAN LENGTHS  8.0000       20.000       12.000
SPAN RATIOS   0.4000       1.0000       0.6000
SPAN RATIOS   0.2000       0.5000       0.3000

  8 ELEMENTS
     ELLGTH        EILEFT       EIRIGHT       RALEFT       RARIGHT
  1   4.000        0.5355E 05   0.5355E 05   0.6667         1.000
  2   4.000        0.5355E 05   0.2678E 06    1.000         1.333
  3   5.000        0.2678E 06   0.5355E 05    1.333         1.000
  4   5.000        0.5355E 05   0.5355E 05    1.000        0.6667
  5   5.000        0.5355E 05   0.5355E 05   0.6667         1.000
  6   5.000        0.5355E 05   0.2678E 06    1.000         1.333
  7   6.000        0.2678E 06   0.5355E 05    1.333         1.000
  8   6.000        0.5355E 05   0.5355E 05    1.000        0.6667

AVERAGE EI FOR EACH SPAN
  0.1071E 06  0.1071E 06  0.1071E 06
AVERAGE EI FOR BRIDGE= 0.1071E 06

AVERAGE RA FOR EACH SPAN
   1.000       1.000       1.000
AVERAGE RA FOR BRIDGE=  1.000

MODE 1: NATURAL FREQ =  2.001


MODE 1 SHAPE,  DSPLS AT 1/8 POINTS

SPAN 1       0.000     -0.043     -0.082     -0.112     -0.130     -0.131     -0.112     -0.070      0.000

SPAN 2       0.000      0.283      0.618      0.885      0.999      0.923      0.678      0.333     -0.000

SPAN 3      -0.000     -0.146     -0.243     -0.292     -0.296     -0.260     -0.192     -0.102      0.000



STEPLENGTH=  0.900       P=  0.1800      LOGDEC(1)=0.0300

MODE 1   F= 2.00  VELOCITY=  1.801
MAX DSPL= 0.4238E-02 AND MAX ACCR= 0.6697      OCCUR ON SPAN 2 AT X=  10.26      WHEN PEDESTRIAN IS AT DIST=  16.65
                                                        (DISTANCES MEASURED FROM LEFT HAND SUPPORT OF SPAN 2)

MAX ALLOWABLE ACCR = 0.5*SQRT(F) = 0.7072      METRES/SECSQD
*************************************************************************************************************************
```

Fig. 3 TYPICAL RESULTS

```
5-SPAN  EIREL=.2 RAREL=.5  LMAIN=20  .4:.6:1:.6:.4
NSPANS=  5
SPAN LENGTHS  8.0000       12.000       20.000       12.000       8.0000
SPAN RATIOS   0.4000       0.6000       1.0000       0.6000       0.4000
SPAN RATIOS   0.1333       0.2000       0.3333       0.2000       0.1333

 16 ELEMENTS
     ELLGTH          EILEFT       EIRIGHT       RALEFT        RARIGHT
  1   4.000          0.5420E 05   0.5420E 05   0.6667          1.000
  2   4.000          0.5420E 05   0.2710E 06    1.000          1.333
  3   3.000          0.2710E 06   0.5420E 05    1.333          1.000
  4   3.000          0.5420E 05   0.5420E 05    1.000         0.6667
  5   3.000          0.5420E 05   0.5420E 05   0.6667          1.000
  6   3.000          0.5420E 05   0.2710E 06    1.000          1.333
  7   5.000          0.2710E 06   0.5420E 05    1.333          1.000
  8   5.000          0.5420E 05   0.5420E 05    1.000         0.6667
  9   5.000          0.5420E 05   0.5420E 05   0.6667          1.000
 10   5.000          0.5420E 05   0.2710E 06    1.000          1.333
 11   3.000          0.2710E 06   0.5420E 05    1.333          1.000
 12   3.000          0.5420E 05   0.5420E 05    1.000         0.6667
 13   3.000          0.5420E 05   0.5420E 05   0.6667          1.000
 14   3.000          0.5420E 05   0.2710E 06    1.000          1.333
 15   4.000          0.2710E 06   0.5420E 05    1.333          1.000
 16   4.000          0.5420E 05   0.5420E 05    1.000         0.6667

AVERAGE EI FOR EACH SPAN
  0.1084E 06  0.1084E 06  0.1084E 06  0.1084E 06  0.1084E 06
AVERAGE EI FOR BRIDGE= 0.1084E 06

AVERAGE RA FOR EACH SPAN
   1.000        1.000        1.000        1.000        1.000
AVERAGE RA FOR BRIDGE=  1.000

MODE 1: NATURAL FREQ =  2.000


MODE 1 SHAPE,  DSPLS AT 1/8 POINTS

SPAN 1        0.000      0.014      0.027      0.037      0.043      0.044      0.037      0.023      0.000

SPAN 2        0.000     -0.051     -0.111     -0.167     -0.207     -0.219     -0.194     -0.123      0.000

SPAN 3        0.000      0.310      0.650      0.906      1.000      0.906      0.650      0.310      0.000

SPAN 4        0.000     -0.123     -0.194     -0.219     -0.207     -0.167     -0.111     -0.051      0.000

SPAN 5        0.000      0.023      0.037      0.044      0.043      0.037      0.027      0.014      0.000



STEPLENGTH=  0.900       P=  0.1800       LOGDEC(1)=0.0300


MODE 1   F= 2.00  VELOCITY=  1.800
MAX DSPL= 0.4299E-02 AND MAX ACCR= 0.6785       OCCUR ON SPAN 3 AT X=  10.00       WHEN PEDESTRIAN IS AT DIST=  16.65
                                                                   (DISTANCES MEASURED FROM LEFT HAND SUPPORT OF SPAN 3)

MAX ALLOWABLE ACCR = 0.5*SQRT(F) = 0.7070      METRES/SECSQD



STEPLENGTH=  0.900       P=  0.1800       LOGDEC(1)=0.0500


MODE 1   F= 2.00  VELOCITY=  1.800
MAX DSPL= 0.3697E-02 AND MAX ACCR= 0.5835       OCCUR ON SPAN 3 AT X=  10.00       WHEN PEDESTRIAN IS AT DIST=  15.75
                                                                   (DISTANCES MEASURED FROM LEFT HAND SUPPORT OF SPAN 3)

MAX ALLOWABLE ACCR = 0.5*SQRT(F) = 0.7070      METRES/SECSQD
************************************************************************************************************
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Fig. 3(cont.) TYPICAL RESULTS

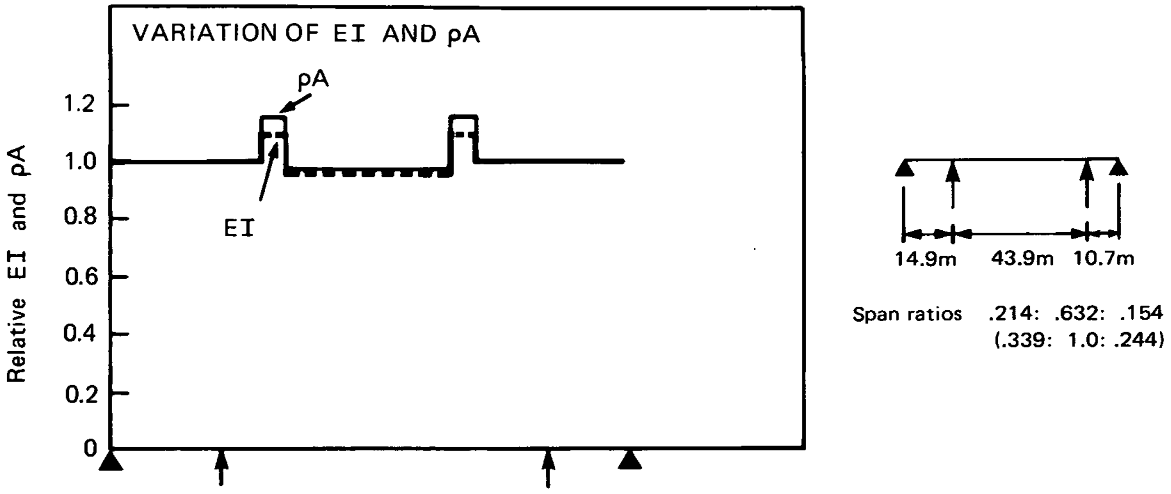


Fig. 4(a) AYR FOOTBRIDGE, Continuous structure, steel box and steel deck

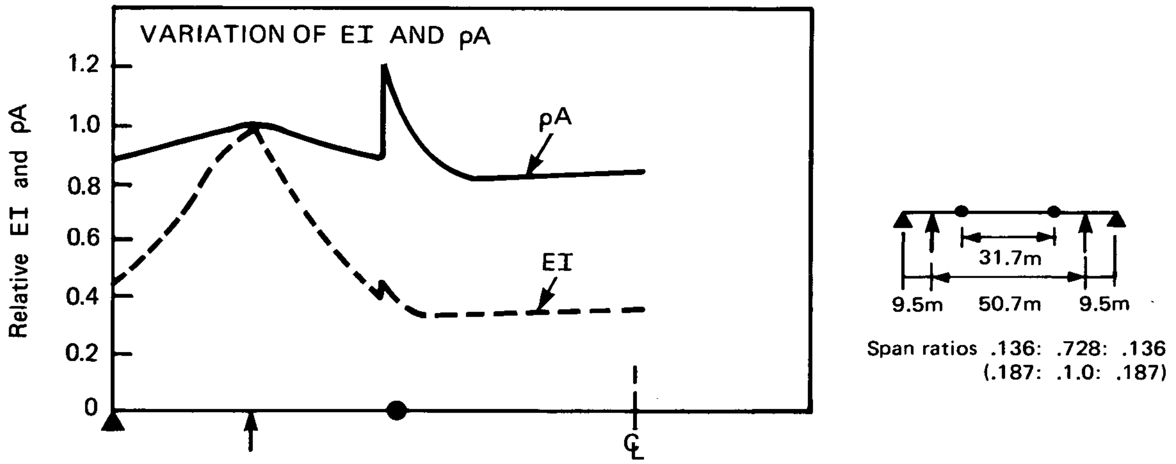


Fig. 4(b) WETHERBY FOOTBRIDGE, Cantilever and suspended span structure, steel box and steel deck

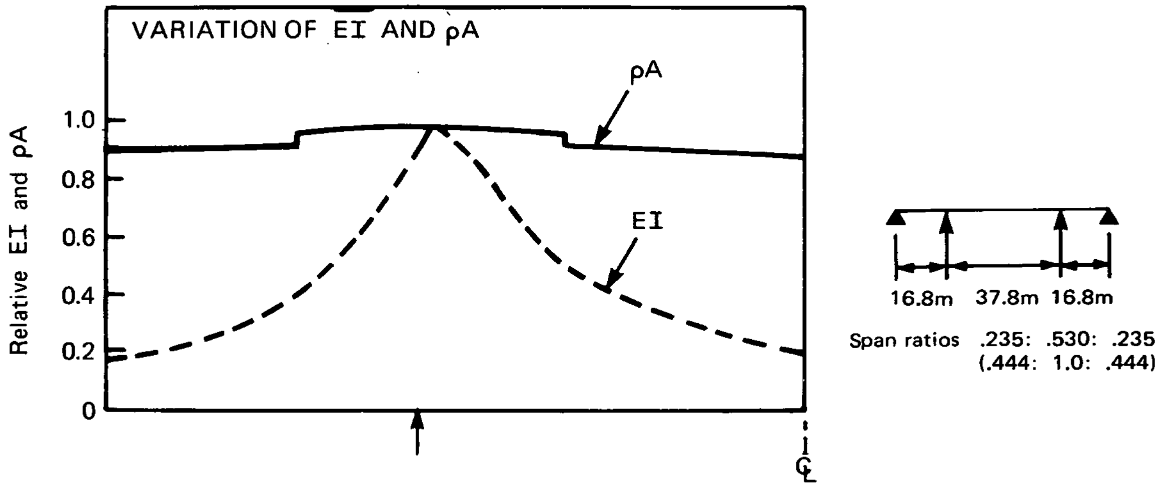


Fig. 4(c) BURY NEW ROAD FOOTBRIDGE, Continuous structure, open and closed steel box acting compositely with reinforced concrete deck

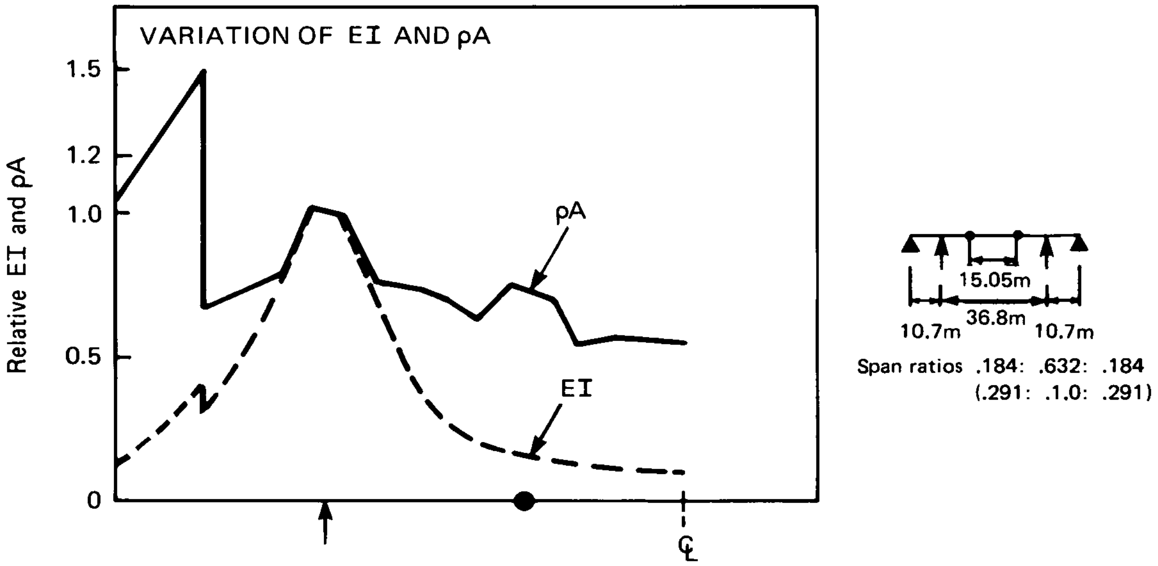


Fig. 4(d) **BRACKENDALE ROAD FOOTBRIDGE**, In situ reinforced concrete, part voided, cantilever and suspended span

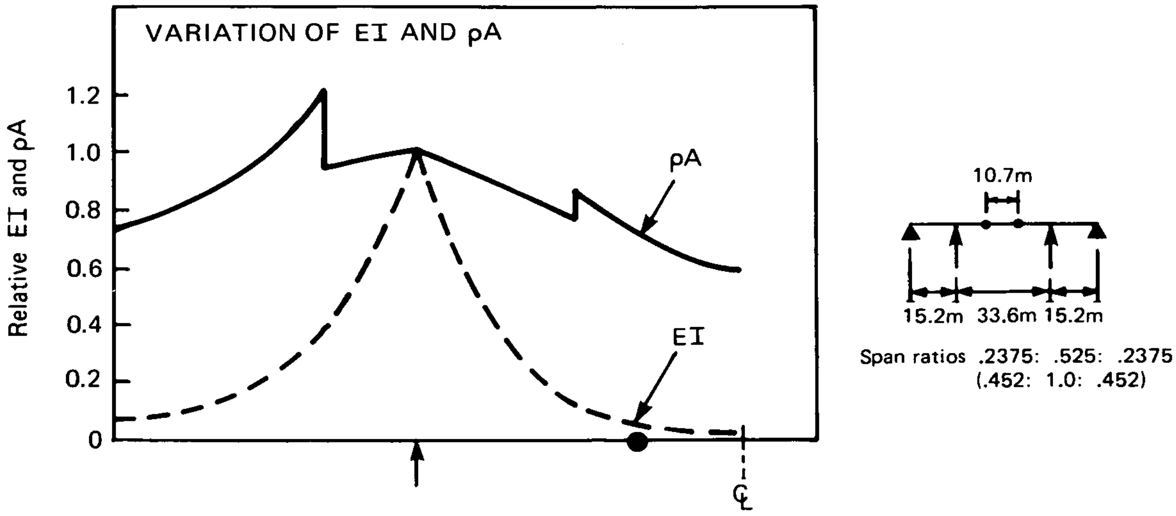


Fig. 4(e) **COTTESMORE FOOTBRIDGE**, Post tensioned concrete, part voided, cantilever and suspended span

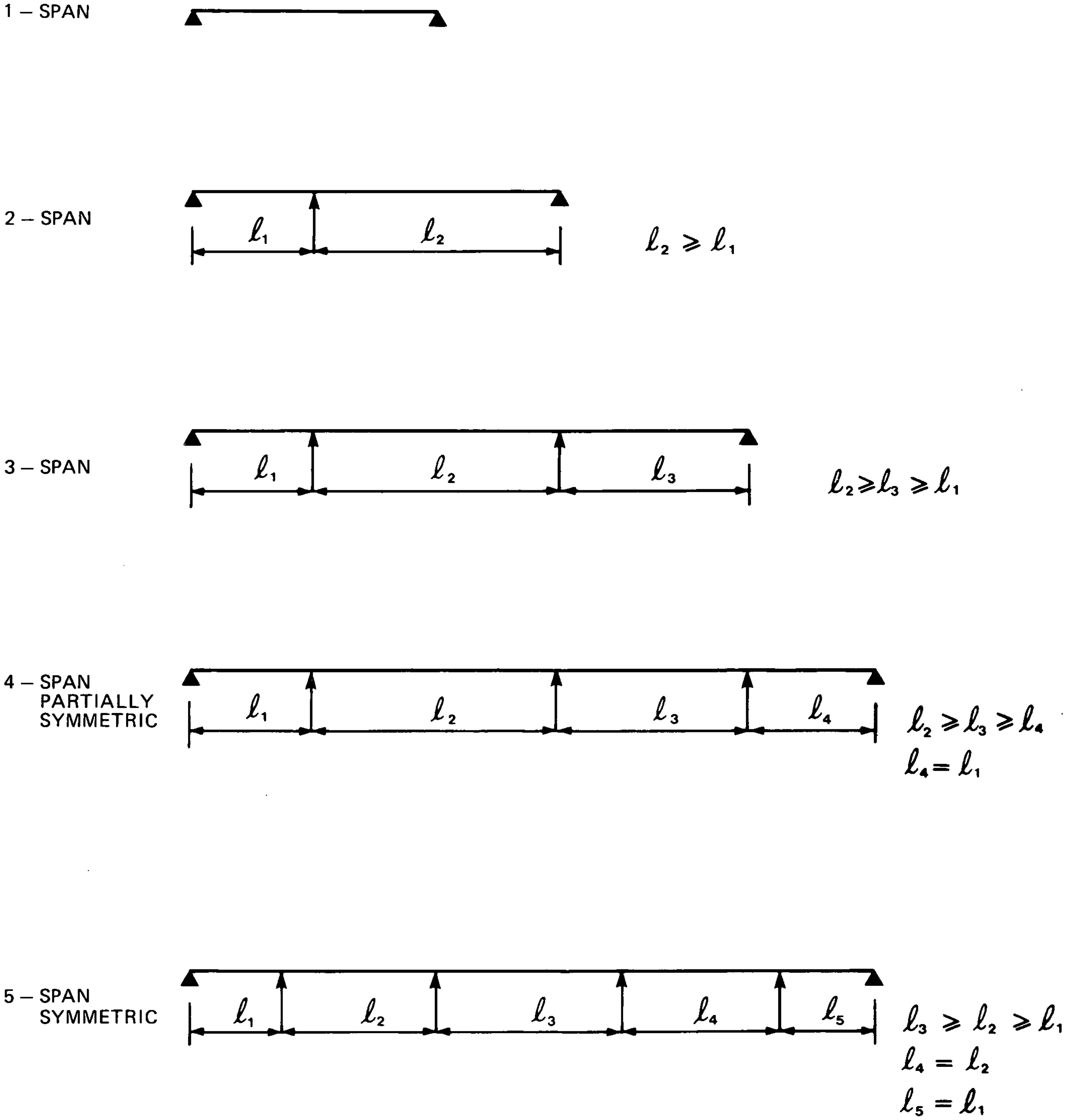


Fig. 5 SPAN ARRANGEMENTS CONSIDERED

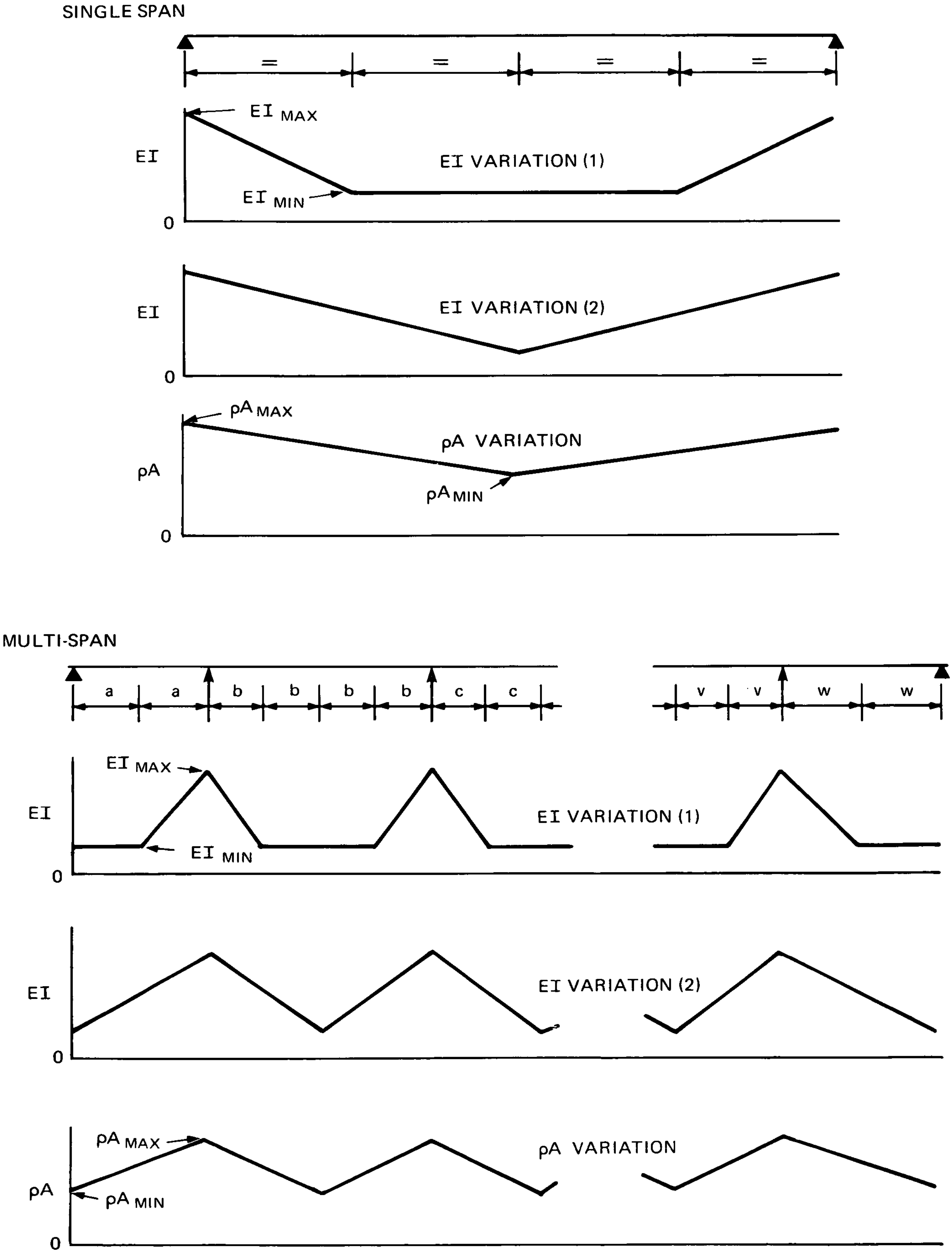


Fig. 6 PRINCIPAL VARIATIONS OF EI AND ρA CONSIDERED

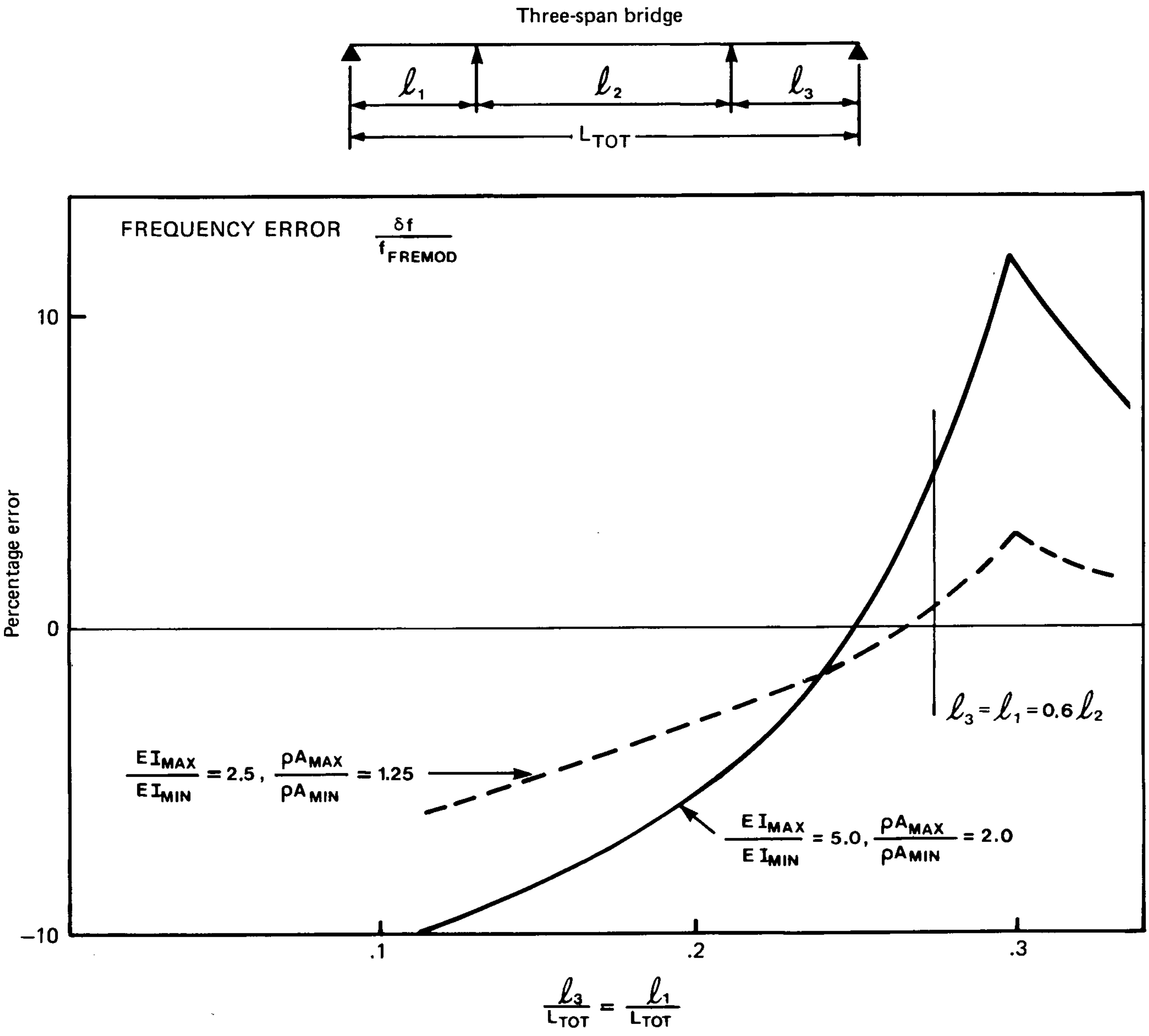


Fig. 7 TYPICAL ERRORS

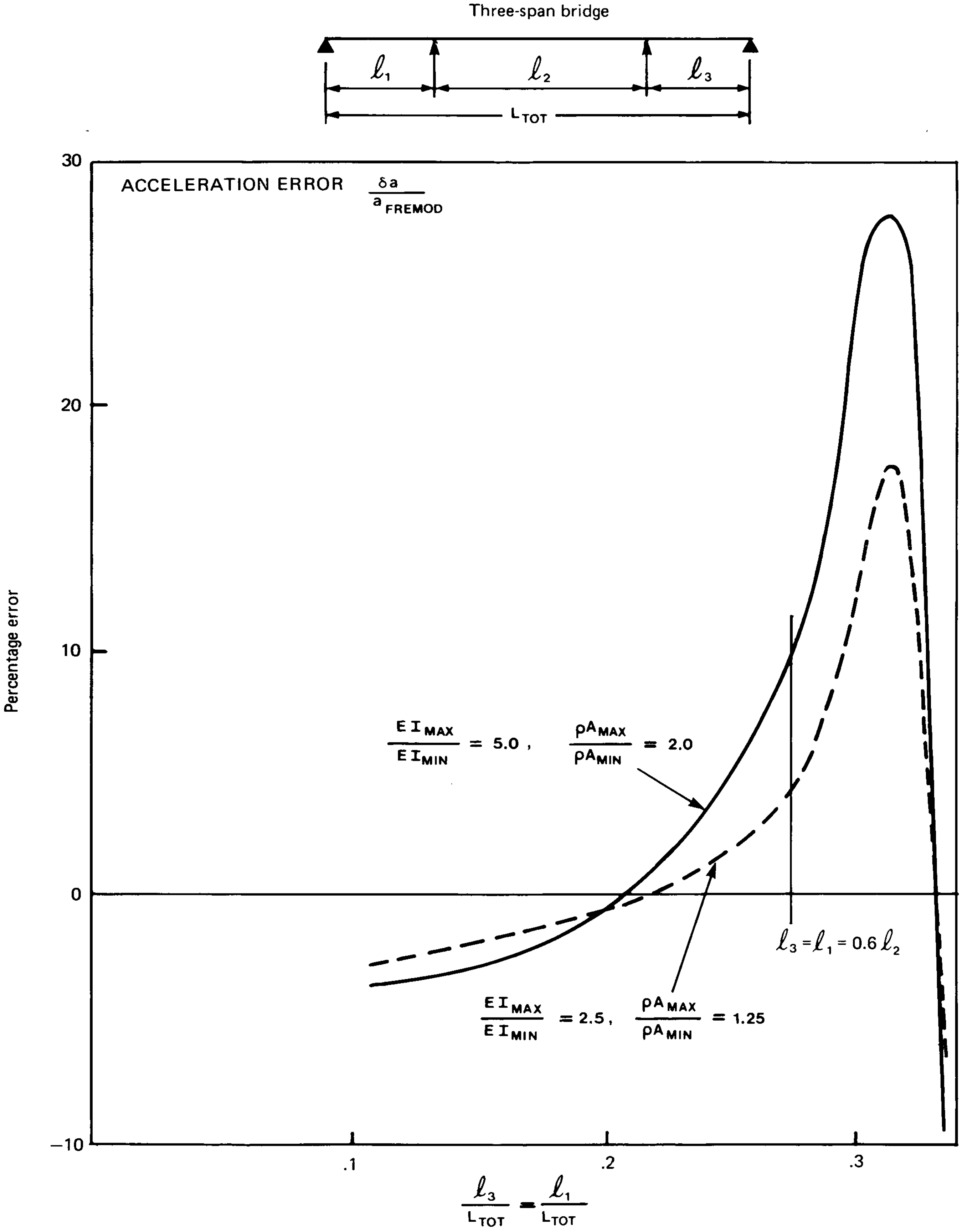


Fig. 7(cont.) TYPICAL ERRORS

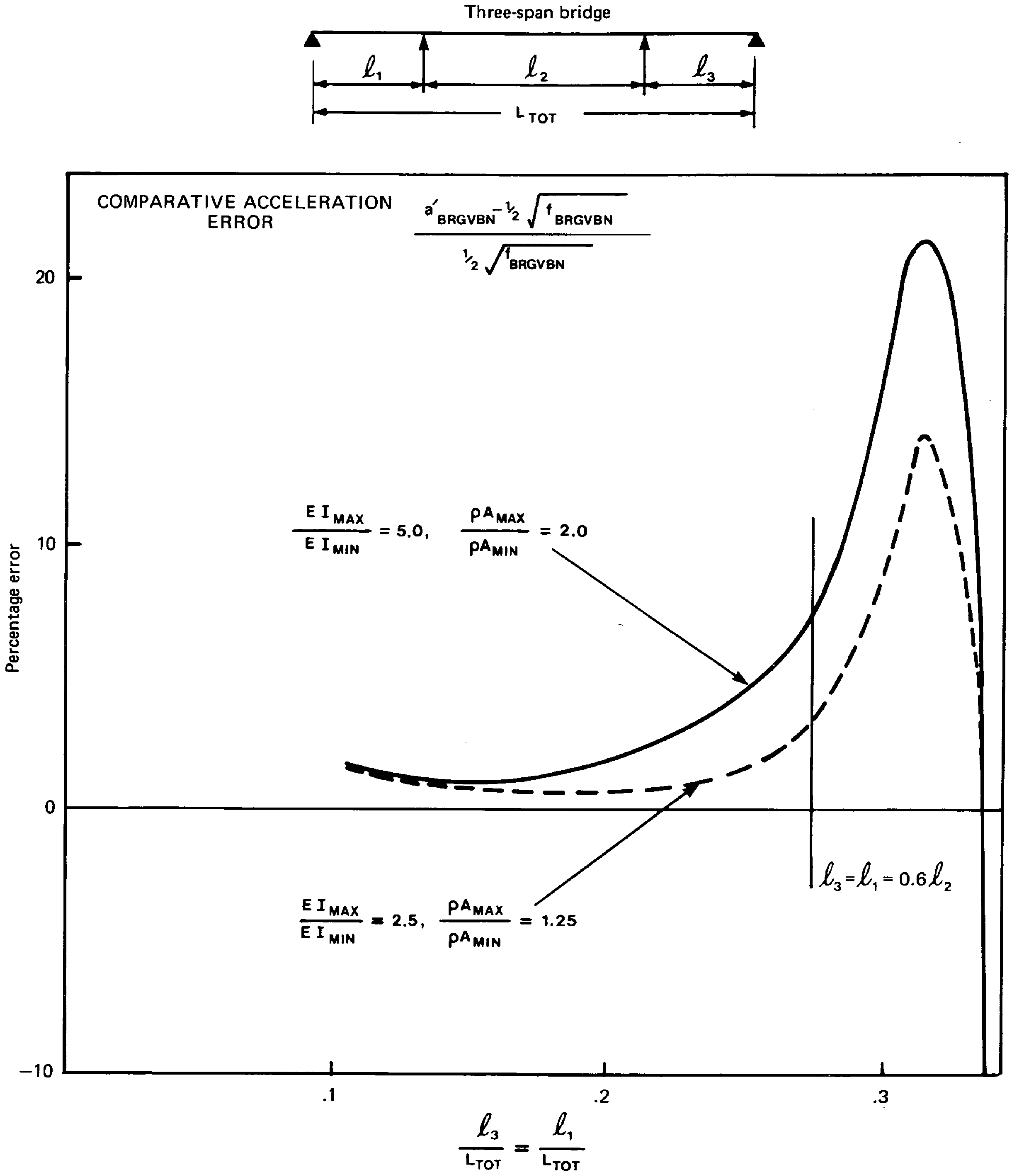
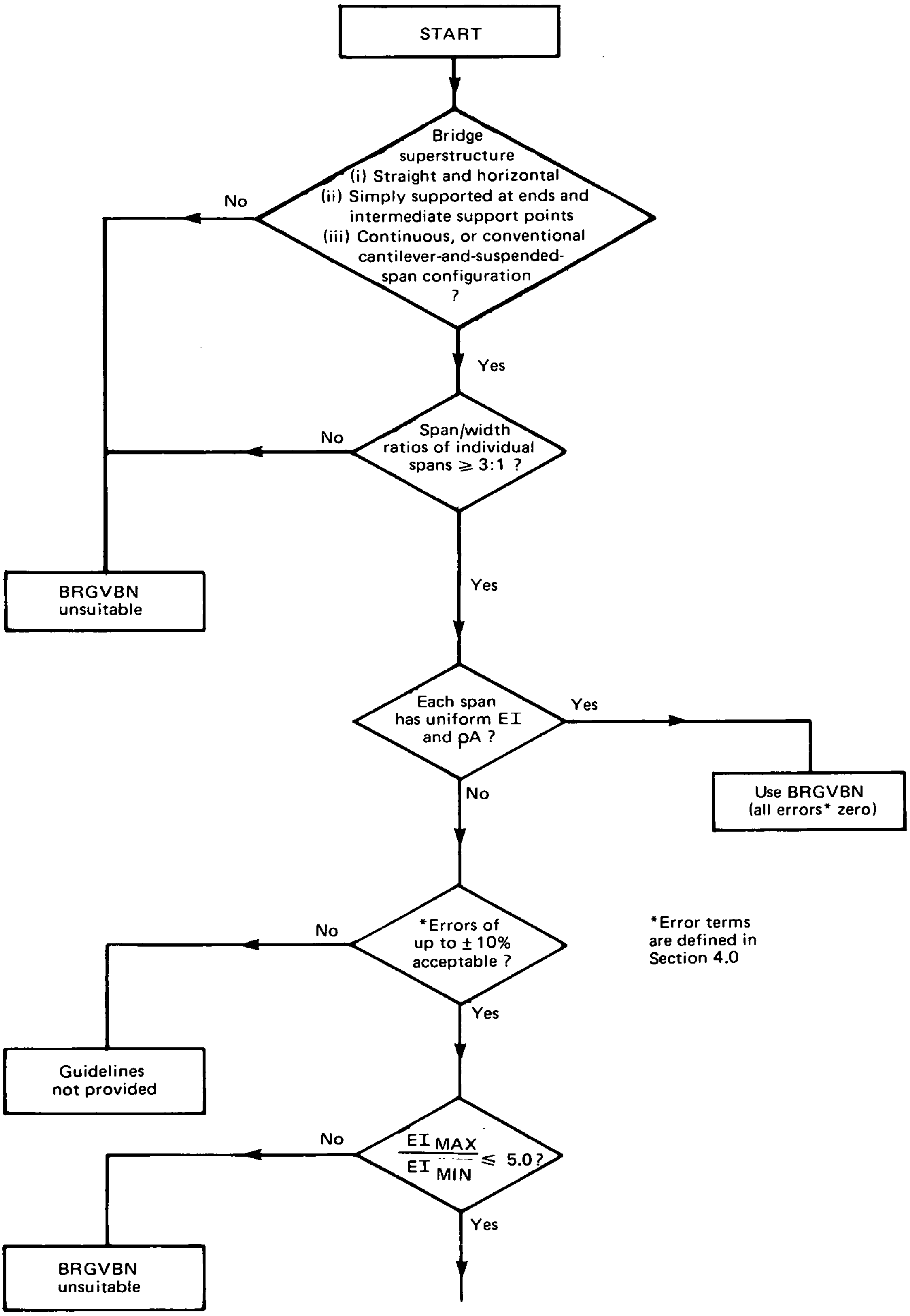


Fig. 7(cont.) TYPICAL ERRORS



*Error terms are defined in Section 4.0

(cont.)

Fig.8 GUIDELINES SHOWING RANGE OF APPLICABILITY OF BRGVBN

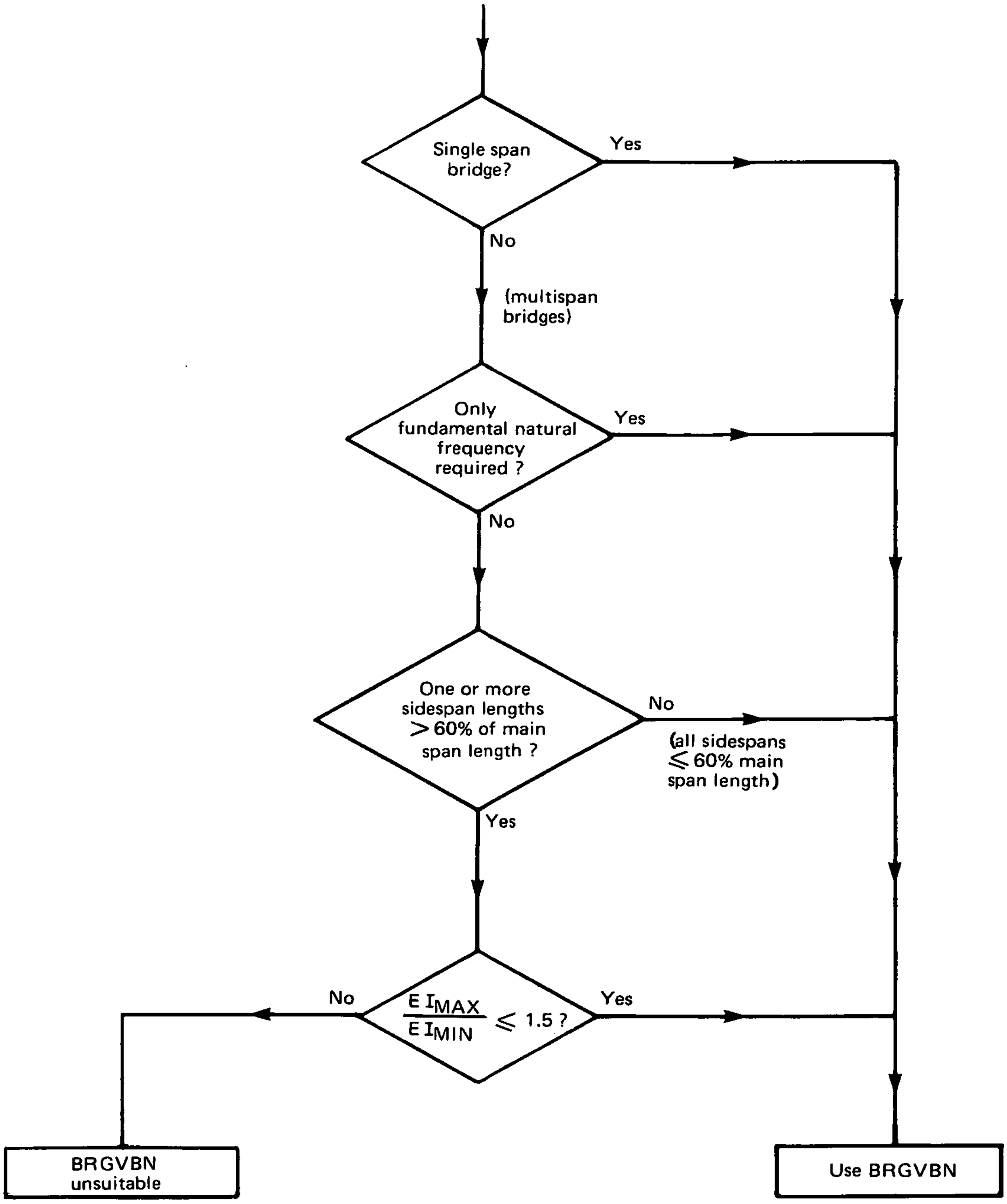


Fig. 8 (cont.) GUIDELINES SHOWING RANGE OF APPLICABILITY OF BRGVBN

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ABSTRACT

A SIMPLIFIED METHOD TO CALCULATE DYNAMIC BEHAVIOUR OF FOOTBRIDGES: *J Wills:* Department of the Environment Department of Transport, TRRL Supplementary Report 432: Crowthorne, 1978 (Transport and Road Research Laboratory). A simple computer program to calculate the dynamic behaviour of footbridges is described. In accordance with recent design rules the fundamental natural frequency of the bridge superstructure and the acceleration response due to the passage of a pedestrian can be calculated. The accuracy of the program has been assessed by extensive comparisons with a much larger 'beam-element' program and with experimental measurements. Guidelines are given defining the range of applicability of the program and their use is demonstrated by reference to eleven existing bridges considered in earlier work. The fundamental natural frequency of many types of highway bridges can also be calculated.

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