TRANSPORT RESEARCH LABORATORY

Design of bridge with external prestressing: design example

Prepared for QSCE Division, Highways Agency

A F Daly (TRL) and P Jackson (Gifford and Partners)

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Background

There has been increasing interest in recent years in the construction of post-tensioned concrete bridges with external unbonded tendons. Experience has shown that external post-tensioning can provide an efficient and economic method for bridge design for a wide range of conditions. In spite of this, there is a lack of general information on how the method can be applied and there are no specific guidelines available in the current UK bridge design code. The current interest is primarily in response to the doubts surrounding the durability of the more traditional form of post-tensioned construction, ie, internal tendons contained within ducts and grouted. It is envisaged that tendons placed external to the section would be more accessible for close inspection and replacement of the prestressing system would be facilitated if required.

To assist in the design of externally post-tensioned bridges the Highways Agency issued a Standard, BD 58 (Highways Agency 1994b) and an accompanying Advice Note, BA 58 (Highways Agency 1994a) in 1994. These documents were based on a review of existing knowledge which included recommendations on where further research was required. To support and develop these documents, the Highways Agency commissioned a programme of research at TRL which included the design of a typical externally post-tensioned bridge, and the construction and testing of a quarter scale model bridge.

Scope of the project

The project is concerned with the development of design guidelines for the use of external post-tensioning in the design of bridges. The project was devised to encompass new design as well as the strengthening of existing bridges. The focus of the project was the design and construction of a model bridge which was eventually loaded to failure. The bridge was designed at full scale by Gifford and Partners, who have been involved in the design and construction of a number of externally post-tensioned bridges. They were commissioned to produce a design based on the current design code, BS 5400: Part 4, for concrete bridges as modified by BD 58, without any arbitrary strengths or weaknesses so that the resulting structure was ‘tight’ to the code requirements. The bridge was subsequently built at quarter scale at the Transport Research Laboratory. The purpose of the project was to examine the procedures used to design the full scale bridge, monitor the construction of the model, determine its behaviour when loaded to collapse in flexure and recommend whether changes should be made to the standards pertaining to the design of bridges of this type.

The construction of the model, the load test and its conclusions are described in a separate report (Woodward and Daly 1999). To provide further assistance to designers, it was considered appropriate to make details of the design available as an example which could be used to illustrate the existing standards. The design example, along with some sample calculations, are presented in this report.

Summary of the report

The purpose of this report is to present an outline design of an externally post-tensioned bridge to illustrate the concept and to present sample calculations, which can be used in conjunction with the existing standards. The bridge considered is an externally prestressed box girder structure, representing a highway bridge with two spans, 32m and 48m, carrying a two-lane carriageway over a motorway. The report contains a description of the main features of the bridge and describes how the concept design was devised. It describes the principles involved in the analysis appropriate for this type of construction and how these differ from the analysis of conventionally post-tensioned bridges. Detailed calculations for shear and flexural calculations are included in the report.

An outline design for an equivalent internally prestressed bridge is also given and both structures are costed. The internally prestressed bridge is found to be marginally cheaper when initial costs only are considered. However, other factors could be considered to outweigh this. These include the costs of maintenance, inspection and potential strengthening, as well as traffic delay costs associated with these activities.

Conclusions

It is intended that this report be used in conjunction with BD 58 for the design of externally post-tensioned bridges. The report outlines concepts required for the analysis and gives sample calculations for determining flexural and shear strength. The report also compares the costs of an externally post-tensioned bridge with a conventional structure. The conclusion is that, while there is a minor cost penalty for this particular structure, this can be out-weighed by considerable savings when whole life costs are considered.

1 Introduction

Gifford and Partners were commissioned by TRL to design an externally post-tensioned bridge. The design was produced as part of a Highways Agency research project investigating the behaviour of externally post-tensioned bridges which included the load testing of a model bridge. The construction of the model, the test to failure and the conclusions arising from the load test are described in a separate report (Woodward and Daly 1999). The present report gives detailed design calculations illustrating the combined use of BD 58 (Highways Agency 1994b) and BS 5400: Part 4 (British Standards Institution 1990) for the design of this type of bridge.

The bridge considered is an externally prestressed two-span continuous in-situ box girder bridge. Detailed calculations for shear and flexure are included in the report. An outline design for an equivalent internally prestressed bridge is also given and both structures are costed. The internally prestressed bridge is found to be marginally cheaper. However, other factors could be considered to outweigh this.

2 Concept design

The span arrangement and section of the bridge are illustrated in Figure 1 and 2 respectively. Some features of the design, notably the overall length and width of the bridge, both of which are close to the minimum likely to be found in practice for this type of structure, were dictated by the requirements of the model test. This was necessary because the bridge was to be tested on a particular test bed at the largest possible scale which was quarter full size. The design is therefore not necessarily the most realistic or economic: it serves primarily to illustrate the various clauses in the code.

The model was to be prestressed with normal commercially available tendons of a realistic type. This necessitated the use of larger tendons than would normally be used in practice. The requirement in clause 2.1 of BD 58 (Highways Agency 1994b) that the prestress should be replaceable without restricting traffic, would make the use of such large tendons impractical unless a prestressing system was used which allowed individual strands to be de-stressed and replaced.

The section used is shown in Figure 2. It was developed from a consideration of practical flange thicknesses combined with the minimum web thickness required to comply with the code requirement for the upper (web crushing) limit in shear. The depth is based on a realistic, but quite shallow, span to depth ratio. This is generally better aesthetically than a deeper section and was desirable for the test to give a highly stressed section. However, the bridge itself would have been cheaper if made deeper, although other factors, such as cost of earthworks, may mean that the overall cost of the scheme would be increased.

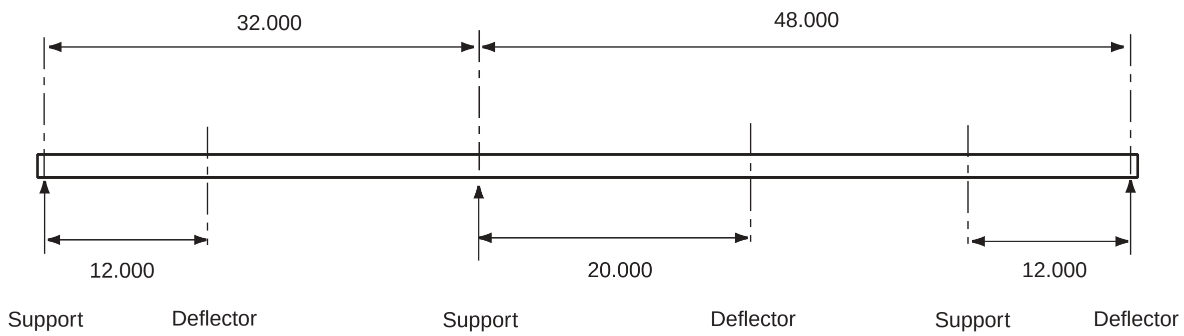

Figure 1 General details of bridge

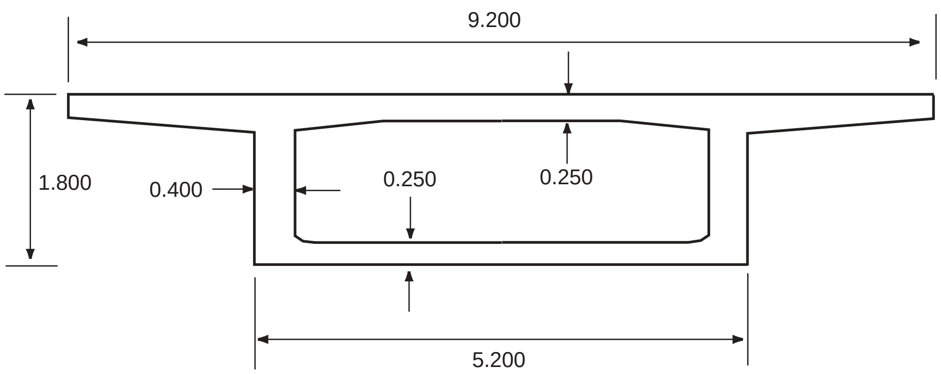

Figure 2 Basic cross-section of bridge

3 Loading and global analysis

As the bridge has only two spans, it was assumed that the whole structure would be cast on falsework in one go. This meant that the dead weight could be simply applied to a computer model of the completed structure. Detailed analysis of construction conditions was therefore not required, unlike in a longer bridge of this type which would be built span by span.

The bridge was designed for normal HA and 45 units of HB load to BD 37 (Highways Agency 1988). It was assumed to carry two lanes of traffic although it is actually narrower than most real two lane bridges.

A simple line beam computer model was used for the analysis using uncracked section properties throughout. In a real bridge, a more detailed computer model would normally be used to give a more accurate analysis of load distribution and shear lag. In the example, the full section was taken to be effective. BS 5400: Part 4, clause 5.3.1.2, always allows this at the ultimate limit state (ULS). At the serviceability limit state (SLS), it specifies that the effective width may be taken as the web width plus one tenth of the distance between the points of zero moments on each side of a web. The points of zero moment may be assumed to be 0.15 times the span from the supports of continuous beams. This would not enable the full width to be taken as effective at the support. However, because the bridge has only two spans, the actual points of contraflexure are further from the supports and, using the actual positions, the use of the full section can be justified.

As the bridge is statically indeterminate, the prestress will induce parasitic moments and it is therefore essential to include them in the computer model. This was achieved by applying the vertical forces at the deviators to the model. The longitudinal force was not applied. This was not required because the line of the neutral axis is straight, the bridge being straight in elevation and having a constant section, so the longitudinal force does not cause moments. For bridges where this does not apply, it is generally more convenient to model the bridge in true geometry and then to apply the horizontal, as well as the vertical, component of the prestress forces. In this way, the model automatically analyses the effect of varying prestress eccentricity due to the variation in position of the neutral axis. Even in a bridge which has a constant section, this can have a significant effect if the highway geometry has a vertical curve.

Once the deviator positions in the main span are assumed, the tendon geometry in this span is defined since it is clearly economic to use the maximum practical eccentricity at support and mid-span diaphragm positions. This may not be the case for the shorter 32m backspan. In order to facilitate the optimisation of the shorter span deflection and the prestress force, the forces due to a unit prestress were applied in two components. The first was that due to a unit prestress force with no backspan deflection, and the second due to a unit prestress force with a unit deflection at this position. The effects were then combined using a spreadsheet program.

4 Design calculations

4.1 Global flexural design

4.1.1 Design of prestress

As the structure is of in-situ form (rather than glued segmental) and has no bonded prestress, BD 58 allows it to be designed to crack under service loads, ie, as partially prestressed. This means that, unlike a conventional structure with bonded prestress which would normally be designed to Class 1 (ie, no tensile stresses permitted in the concrete), there is no unique solution for the prestress required to comply with the code. The minimum allowable prestress is restricted only by the requirement that it remain in compression under permanent loads. This minimum prestress is approximately 31MN before losses. However, this would require a large amount of secondary reinforcement to comply with ULS and crack width criteria. To achieve no-tension under HA Combination 1 load (which would be required for a conventional bonded structure) requires some 48MN prestress. The actual prestress used is likely to be between these two values.

The trade-off between reinforcement and prestress quantities means that an investigation is required to determine the optimum solution. However, because of the practical limitations mentioned in Section 2, it was necessary to use a small number of relatively large cables and the number of possible economic solutions was limited. Three cables each side giving a total jacking force of 37.5MN at full size was obviously the best solution, two being insufficient to avoid tension under permanent load and four being more than enough to avoid cracking under service load.

The spreadsheet program described in Section 3 was used to calculate the stresses at each section based on the gross concrete section. However, the final design is based on cracked section analysis using the BS 5400 reinforced concrete crack width equation. This is required by BD 58 primarily because the ‘hypothetical tensile stresses’ used in Class 3 design to BS 5400 were derived assuming bonded prestress. The use of cracked section analysis gives greater freedom to the designer at the expense of more calculation.

Because the final design uses cracked section analysis, the spreadsheet can only give an indication of the required prestress geometry and force. The final section analysis has to be done separately and this was undertaken using the commercial program SAM (Bestech Systems 1994). Minor alterations to the prestress geometry could be made following an initial section check. In the event, the only change made was a slight increase in the backspan deflection to avoid the need for additional secondary steel there.

The stresses given by the spreadsheet for the final design, after losses have been taken into account, are shown in Figure 3. The losses were estimated in the conventional way, the only difference from the calculations for a bonded section being that the creep is calculated from the average stress at tendon level, rather than from the stress at tendon level at a particular section. In the event, the stress at tendon level varied little and was quite low, as will be seen from Figure 4 which shows the permanent stress state. This meant that the creep losses were relatively small and easily calculated.

It can be seen from Figure 3 that the tensile stresses calculated on the gross concrete section are very significant particularly in the soffit of the main span. This indicates that, unlike in a conventional bonded prestress design, the concrete is designed to crack under service load. The tensile stresses shown in Figure 3 are not real: they are the hypothetical stresses which would exist if the

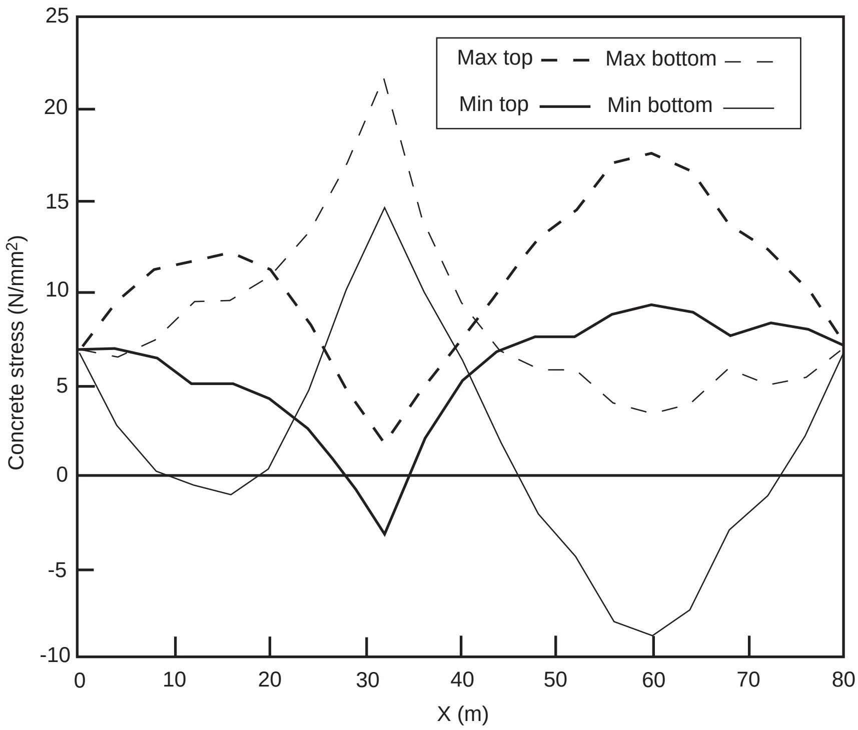

Figure 3 Worst stresses on gross concrete section
(Calculated with prestress after losses for load combination 1 HA and HB)

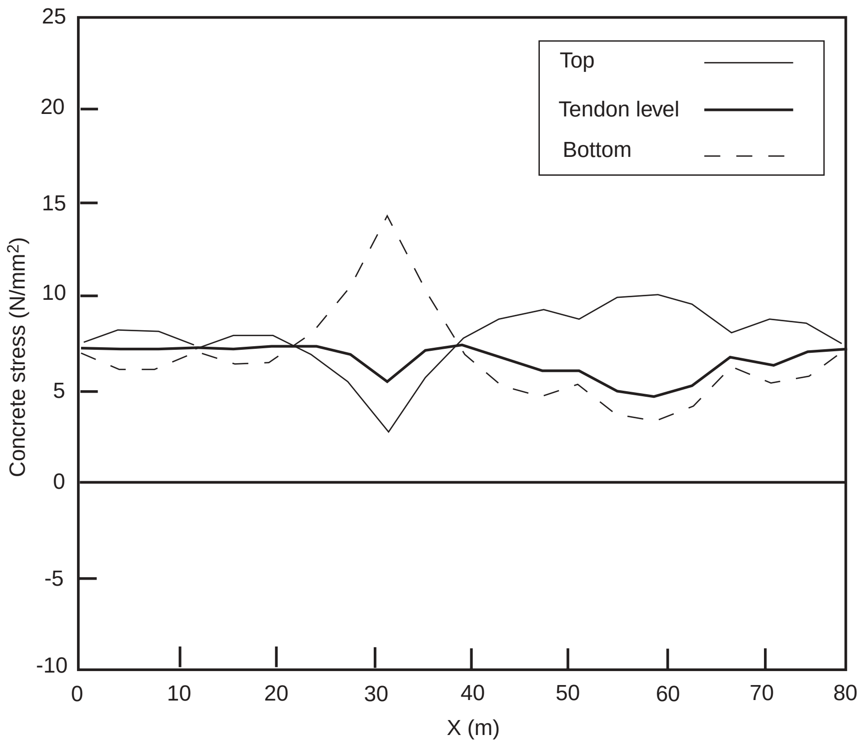

Figure 4 Permanent stresses
(Calulated with prestress before long term losses)

section did not crack. The final section check is done using a cracked section analysis but, as in conventional reinforced concrete design, the global analysis of the structure can still be done using gross concrete properties. It will be seen in the next section that this design requires a very significant amount of secondary reinforcement (a maximum, at full scale, of alternate 32 and 40mm bars at 125mm centres) and it is likely that in a real design a slightly higher prestress force would be used to reduce this requirement, as explained in Section 1. This option was not available as the design was constrained by the requirements of the model. However, the design adopted is both allowable and economic under the code.

Being effectively a partially prestressed bridge, this design represents a significant departure from previous UK bridge design practice. This was one reason for testing this form of structure, particularly as previous comparable tests on externally prestressed model bridges have been on structures at the opposite extreme, ie, fully prestressed glued segmental structures with no continuous secondary steel (MacGregor *et al* 1989, Muller and Gauthier 1990).

4.1.2 Ultimate flexural strength and design of reinforcement

Unlike in a conventional bonded prestressed design, ultimate strength was considered first as it was anticipated (correctly) that it would be critical. The design was undertaken using SAM section analyses and the plot from a spreadsheet is shown in Figure 5. This gives the envelope (maximum and minimum values) of the net bending moment on the section, that is the applied moment minus the moment due to prestress. The capacity of the section with only nominal reinforcement (taken to be T12-250 on all faces) is also shown in the plot. This capacity was calculated from a SAM analysis of the section with the design prestress force treated as an applied axial load. Note that, since the global analysis is based on the gross concrete properties, the axial force should be applied at the elastic neutral axis level and the moment capacity should be calculated about this axis. SAM does this automatically although it uses the neutral axis of the transformed section including the reinforcement, rather than just the concrete section. This means that the direct use of SAM output is marginally conservative when heavy tensile reinforcement is used although the results can easily be corrected.

The nominal T12-250 reinforcement was included in the section analysis as it gives a significant increase in capacity. The design prestress was taken as the prestress calculated after all losses multiplied by γ_{fL} which is 0.87. This approach ignores the increase in tendon force which occurs in reality as ultimate failure approaches but it is required by BD 58 unless a non-linear analysis is undertaken. A small increase in strain (equivalent to 100N/mm^2 increase in stress) is allowed without such analysis for cables at mid-span but not if, as here and in most cases, the cables extend beyond the supports.

The prestress moment could equally well have been applied on the capacity side of the equation, as it would normally be with internal prestress. If this is done, the gross applied moment is considered and the prestress moment, as well as axial force, is included in the section analysis. However, the secondary effects of prestress (the parasitic moments) should still be included and this is usually done by including them on the load side. This means that the gross applied moment is obtained not by ignoring the prestress but by including it and then subtracting the primary moment. With a statically indeterminate structure, it is therefore more convenient to include the prestress on the load side of the equation as here. However, this does mean that the prestress load factor of 0.87 is applied to the parasitic as well as the

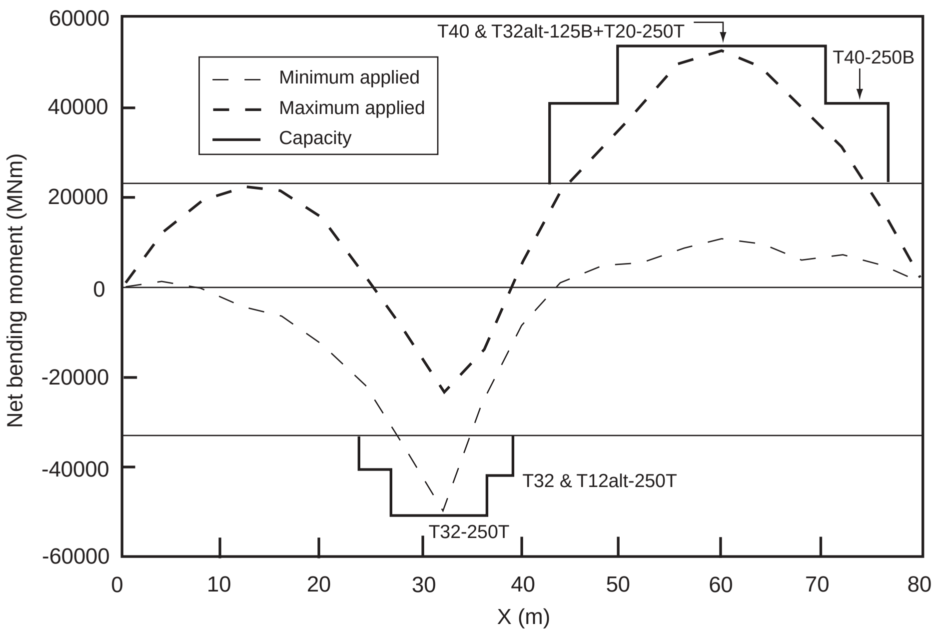

Figure 5 Ultimate moments (Sagging positive)

primary moment which, arguably, is not required by BS 5400. Considering the prestress on the load side, as here, also has the advantage that it means the capacity as plotted in Figure 5 does not vary with prestress position.

Although the ultimate flexural section analysis was done by computer, an example of the hand calculation approach is given in Appendix A.

It can be seen from Figure 5 that, with only nominal secondary steel, there is a significant shortfall of capacity in the critical regions, ie, sagging in the main span and hogging over the support. The capacity is just adequate in the backspan although, as noted previously, this arose only after the deflection of the tendons in the backspan was increased. The shortfall in capacity is made up by adding more secondary steel, replacing some of the nominal T12-250 with heavier reinforcement. The capacity of the section with increased steel was again calculated using the SAM program and the steel area increased until adequate. The capacity with the full increased steel area and a suitable intermediate area was then plotted on Figure 5, in this case by hand, and the curtailment positions decided essentially by eye.

When deciding the curtailment positions, it is important to realise that an envelope of maximum and minimum moments is only valid at a particular position if the critical load cases for that position were analysed. Also, the requirement for additional longitudinal steel for shear has to be included. BD 58 covers this by simply saying that reinforcement or prestress which is anchored within h/2 of the section being considered in flexure should be ignored. In effect steel has to extend h/2 from the point where Figure 5 would suggest it is no longer needed.

The design of reinforcement in Figure 5 is very close to the limit. This was intended for the test bridge. In practice, unless one is very confident that the critical load cases for the curtailments had been included in the analysis, the curtailments would not normally be designed so tightly. However, even for the test bridge, the curtailments are not as tight as they appear in the figure. The design as shown would have required 40mm bars to lap with 12mm bars and this inelegant detail was avoided by including intermediate size transition bars. Their effect on flexural capacity is not shown in Figure 5.

4.1.3 Cracking and service stresses

The critical sections were also checked for crack width and concrete stress, again using SAM, treating the prestress force and moment as part of the applied load. As this is an SLS calculation, the 0.87 γ_{fL} does not have to be applied but the critical case is still the prestress after all losses. The maximum widths were 0.08mm at the support and 0.20mm at mid-span under full combination 1 loading including the 45 units of HB. These figures compare with the allowable value of 0.25mm but this does not have to be checked under the full combination 1 loading so the crack widths were well within the code requirement. It appears that this will normally be the case due to the rather severe ULS check and the acceptance of cracked section analysis at SLS.

The maximum concrete compressive stress given by the SAM section analysis was 22N/mm^2. This is very close to the maximum stress given in Figure 3 but this will not always be the case. Normally, a cracked section analysis gives a slightly higher worst compressive stress than an uncracked analysis such as that used for Figure 3. In this particular case, however, the considerable quantity of secondary reinforcement included in the cracked analysis compensates for this effect. This suggests that a gross concrete analysis such as that considered in Figure 3 gives a good *indication* of the maximum compressive stress, and hence of whether a design complies with the stress limits in BS 5400, but does not give an exact analysis. It should be noted, however, that an internally prestressed Class 3 design to BS 5400 could be outside the stress limit since there is no requirement to check the stresses using a cracked section analysis.

The section adopted and shown in Figure 1 is highly stressed at the support requiring 60N concrete to comply with the SLS compressive stress limit and the upper limit in shear and to avoid an over-reinforced brittle bending compression failure mode. Elsewhere in the structure, 50N concrete would be adequate and the local over-stress could have been avoided by thickening the bottom flange and webs locally over the centre support.

In principle, BS 5400 requires service stresses and crack widths to be checked under load combination 3, which includes differential temperature, as well as load combination 1. An analysis of differential temperature was undertaken for this design but it was found not to be critical.

4.2 Shear

BD 58 requires externally prestressed sections to be treated as reinforced concrete with the prestress considered as an applied load. The component of prestress along the member enhances the shear strength whilst the component perpendicular to the member normally reduces the shear force.

The design for shear was again carried out using a spreadsheet. However, for the purposes of illustrating the code, example hand calculations are given below. In the following, the bridge is treated as a simple beam, and torsion is not considered. In a real design, a more sophisticated analysis would be used and torsion would be considered. The calculations for this would be the same as for a conventional design since BS 5400 does not differentiate between prestressed and reinforced concrete.

4.2.1 Section at the central support

This section has the highest shear force and is therefore critical for the upper web crushing limit. Since this check determines the minimum allowable web thickness, it would normally be done (albeit somewhat approximately) early in the design process to check that the section is adequate.

Gross applied shear	=	9721kN	(with ULS load factor applied)
Prestress shear	=	-1677kN	(after losses and with ULS load factor, 0.87, applied)

The parasitic prestress shear is the vertical component of the prestress force which helps to resist the applied shear force acting at this point. Therefore:

Net shear = 8043kN

Now

Shear stress, v = V/bd

where b is the total web thickness (ie, the two walls) and d is the depth to the tension reinforcement, here taken to be the top layer. Thus:

v = 8043000/2x400x1735
= 5.79N/mm²

Allowable upper limit = 5.8N/mm² for 60N concrete (BD 58)

This confirms that the section is the minimum allowable with 60N concrete. If this were not available, the problem could be overcome with very local thickening of the webs.

The short shear span enhancement (BS 5400: Part 4, clause 5.3.3.3) can be applied to v_c. This becomes infinite immediately adjacent to the support so this section cannot be critical for link design.

4.2.2 Section 4m from central support

This section is likely to be critical for link design as it is approximately 2d from the support and therefore the first section which does not benefit from short shear span enhancement.

Gross applied shear = 8737kN (with ULS load factor applied)

Prestress shear = -1677kN (with ULS load factor, 0.87, applied)

Therefore

Net shear = 7060kN

and

Shear stress, v = V/bd
= 7060000/(2x400x1735)
= 5.09N/mm²

Now

A_s = 20914mm²

Note that the steel included here has to extend an effective depth beyond the section considered and therefore the steel which is stopped at the first curtailment position is not included.

A_s/bd = 20914/(2x400x1735)
= 1.5%

Therefore

v_c = 0.87N/mm²

from BS 5400, Table 8, with f_{cu} taken as 40N/mm². BD 58 lifts the 40N limit to the concrete grade used for the upper limit in shear but it does not do it in the formula for v_c. There is no particular reason for this (research evidence suggests that the formula is valid for concrete strengths up to at least 90N/mm²) but the effect is small.

Now

ξ_s = 0.73 for d = 1735

and the ultimate shear stress corrected for axial force is

= $\xi_s v_c(1+0.05N/A_c)$

from BS 5400, clause 5.5.6, which is invoked by BD 58,

= 0.87x0.73(1+0.05x29.15/4.9)
= 0.82N/mm²

Note that the prestress force used is that after losses and has the 0.87 factor applied.

Therefore

required A_{sv}/s_v = 2x400(5.08 + 0.4 - 0.82) (0.87x460)

from BS 5400, Table 7,
= 9.32mm²/mm

Using two pairs of
T16, required s_v = 86mm

Thus, use T16s at 75mm.

In practice, more sections would be checked so the link spacing could be increased away from the critical area. As with main steel curtailments, it is important to ensure the relevant critical load cases are included in the analysis.

It will be seen from the above that the concrete contribution to the shear strength is small, much smaller than it would have been if the normal prestressed rules were used. There has been concern that normal prestress rules, being essentially empirical and based on tests on beams with bonded internal prestress, could be unsafe with external prestress. Recent research by Clark and Toms (1996) and by TRL (Woodward and Daly 1999) suggests that the BD 58 approach is conservative and the prestressed approach is better. BD 58 may later be amended to reflect this but, as yet, no tests on continuous beams have been reported and these may be the worst case.

5 Deflector and diaphragm design

5.1 Tendon geometry

Having fixed the prestress profile, the tendon geometry was made as simple as possible with the tendons straight in plan for most of the length of the bridge. They might have been made straight for the full length but it was found that the required spacing of the anchors prevented this. Also, if the three anchors had been placed side by side the space left in the anchor block for an access hole would have been very restricted. It was therefore decided to place two anchors side by side in the top of the diaphragm and one below. This also has the advantage of positioning the anchors close to the top and bottom flange reducing the amount of reinforcement required to enable the diaphragm to transmit the prestress force to the flanges. The reason for placing the lower anchor immediately below the outer upper one, rather than below the space between the two upper anchors, is for ease of detailing the reinforcement which is still quite congested.

The cable geometry used enabled the deviators in the main span to be of the simple triangular form used for Wadebridge (Hollinghurst 1995). However, the design required the tendon centroid position at the back span deviator to be much higher which would have required a bigger deviator. It proved much simpler to deviate only the outer two cables leaving the inner one straight. This enabled all three deviators to have a similar shape although different reinforcement was required for the back span deviator. Strictly speaking, compliance with BD 58 would have required some nominal restraint for the undeflected cable in the back span as the free length is longer than is normally allowed. The reasons for the restriction are twofold. Firstly, in the ultimate condition, the deflection of the beam between the deviators, where the tendon position is fixed relative to the bridge, can be significant and this reduces the effective eccentricity. Secondly, the first natural frequency of vibration in the free length of the tendon can be so low that it suffers from resonance with vibration of the bridge. BD 58 allows the free length to be longer if checks for these effects are undertaken.

The deviation at the deflectors is achieved by using steel tubes with the HDPE ducts passing through inside so that they are continuous, rather than having to be connected to the steel tubes. In Wadebridge, standard plane structural hollow sections were used. This demands very accurate fixing and fabrication to ensure that the deviator tubes line up with the tendons. Many other structures have used trumpet ends on the tubes to increase the directional tolerance. A solution which makes both the fabrication and fixing easier than this is to use plane tubes recessed slightly into the concrete and then to cast short trumpet ends in the concrete.

The minimum radius of the tendons at the deviators is given by BD 58. In principle, this fixes the minimum thickness of deviators for any given deviation angle. In practice, even with the bigger than normal tendons used in this design, this proved not to be critical as the requisite angle of deviation could easily be accommodated in the thinnest practical deviator.

5.2 Anchor blocks

The design of the local bursting reinforcement around the actual anchors is exactly the same as for a conventional bonded design. However, whereas conventional anchor blocks transmit the anchor forces directly into the concrete section behind the anchorage, anchor blocks in external designs, by definition, have free space on this side. This means they have to span between webs and flanges supporting the prestress forces. The result is that they have to be very heavily reinforced. Excessively crowded reinforcement in the anchor blocks, with consequent difficulty of fixing the reinforcement and compacting the concrete, has been a significant problem on many externally prestressed bridges. In this design the problem was reduced by adjusting the tendon geometry to optimise the position of the anchors as noted in section 5.1. It was also decided to increase the thickness of the anchor blocks above the minimum required which is fixed by the crushing limit in shear. As the anchor blocks in this type of structure are immediately over the bearings, the resultant extra weight is not a problem.

The anchor block is essentially designed for flexure like any other reinforced concrete structure. The design load specified by BD 58 is the characteristic strength of the tendons. The block primarily spans simply supported between top and bottom flange, the benefit from end fixity being limited by the fact that the web and flanges are much thinner than the diaphragm. Shear is also checked, the advantage from short shear span enhancement being very important. Finite element analyses are sometimes used but, in fact, conventional finite element analysis is less realistic for this element than a simple strut and tie analysis.

Having calculated the required reinforcement quantities, the detailing is critical if the cage is to be easy to fabricate. In this design, the flexural reinforcement in the deviator is made up of U-bars which lap on to smaller U-bars to form the bursting reinforcement. This gives a much simpler cage than using totally separate bursting steel.

As with any post-tensioned design, it is important to consider the practical aspects of prestressing. In externally prestressed designs to BD 58, it is necessary to consider possible later re-stressing as well as initial stressing. To facilitate this, the extension and jacking length of the tendons must be left in place and protected from corrosion. This is usually done using wax in a steel enclosure. With the long tendons often used with external prestress, the extension can be a metre or more and the space required to install and remove the covers can be longer than that required to remove the jack.

In this particular bridge, all the tendons are anchored at the ends. In a longer bridge where the tendons are not full length, there would normally be some anchor blocks where tendons from both directions are anchored in the same deviator. Considerable bending moments and shears can be developed. When designing deviators, it is important to realise that the temporary condition, when some or all the tendons on one side are not stressed, is likely to be critical and must be checked.

5.3 Support diaphragm at pier

This is the same as in a conventional design except that it also acts as a deviator. Like the anchors, the deviators have to be designed for the characteristic strength of the tendon. Unlike the anchors, which take the whole of this force spanning between the flanges, they only have to take the force of the deviation angle and transmit it into the rest of the structure. In this bridge, the force due to the deviation is a simple vertical force which essentially goes straight into the bearings. However, because the tendons at this section are inside the bearings, the diaphragm has to be designed to span transversely.

5.4 Span deviators

These deviators could be designed simply to cantilever from the webs. However, the forces and moments involved are sufficient to cause significant stresses due to distortion of the box as a whole. A three dimensional finite element analysis of this region was therefore used in the design of Wadebridge. This was not done in this design, it being judged that the secondary reinforcement is sufficient to resist these forces.

6 Equivalent conventional design

In order to obtain comparisons with a conventional design, an outline design of an equivalent structure with conventional internal bonded tendons was developed. Like the externally prestressed design, this used full length profiled cables. In order to provide space for the tendons in the webs, this necessitated the use of significantly thicker webs, giving the section shown in Figure 6. The combination of the thicker webs and the greater prestress eccentricity possible with the internal tendons for this design resulted in the maximum concrete stress being significantly lower than in the externally prestressed design. The concrete section for the externally prestressed design was the minimum possible within the code without using a varying section, whereas the internally prestressed section could have been reduced without the stresses going outside the code limits. It might be thought that this would distort the cost comparison and it would be better to reduce the section to make the two designs on the limit to the code and therefore more directly comparable. However, the thicknesses of the section had been chosen for practical reasons and it was considered undesirable to reduce them. The concrete stresses in the internally prestressed design could therefore only be made on the limit by reducing the overall depth. This would have increased the prestress required and the net effect would have been to increase the cost. This does, however, indicate that a marginally longer span to depth ratio can be achieved with internal prestress.

The comparison of the designs is slightly distorted by the fact that the externally prestressed design was affected by the requirement to model it at quarter size. The only significant effect of this was in the adoption of a smaller number of larger cables than would be used in practice. This had two effects. Firstly it prevented a precise optimisation of reducing prestress against increasing the requirement for secondary reinforcement. The second effect was that it gave a marginally smaller eccentricity.

The option of reducing prestress and increasing reinforcement is not available in a conventional internally prestressed design to BS 5400: Part 4 and BD 24 (Highways Agency 1992). Being restricted to using large cables which could be provided at quarter scale could, therefore, have had a much greater effect on the economics of the conventional design. It was therefore designed at full scale with more realistic size cables.

The prestress force required at the critical section after losses is some 43MN compared with 33.5MN provided in the externally prestressed design. The higher permanent concrete stresses in the relevant parts of the structure result in the long term prestress losses being greater. The higher friction losses result in the actual jacking force required being some 58MN compared with 37.5MN for the externally prestressed design. The benefit of reduced friction losses in this structure is not as great as in many others due to the fact that the critical section is in the main span and relatively close to the jacking position. A previous exercise using an otherwise similar bridge with two equal spans, had shown a gain of some 20% compared with only 8% in the design considered here. This is due to the critical section being over the support giving over double the friction losses to the critical section.

7 Economics

7.1 Quantities and initial costs

The quantities and cost estimations for the two deck designs is shown in Table 1. Only the basic deck structure is included. The cost of substructure, surfacing, waterproofing and deck furniture as well as falsework is assumed to be similar for the two structures. The total cost quoted is therefore significantly less than the real total cost which makes the price difference between the two designs greater when expressed as a percentage of the total cost.

The saving of prestress in the unbonded design is partly offset by the higher unit cost of the prestressing systems.

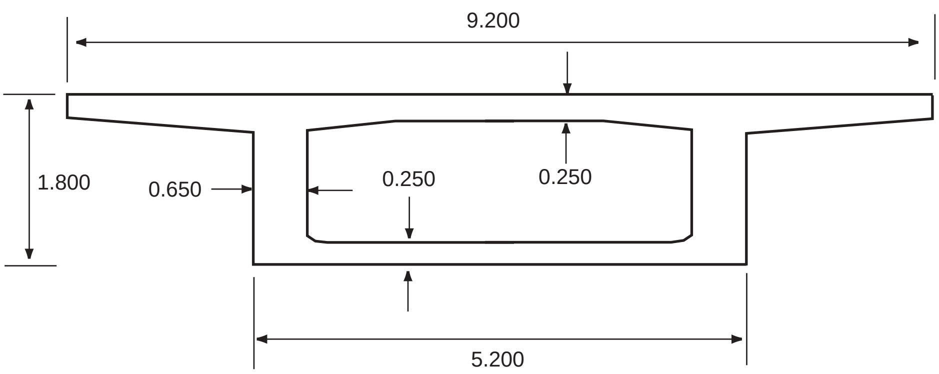

Figure 6 Section with internal tendons

Table 1 Comparison of quantities

Item	*Type*	*Unit*	*Rate*	*External prestress*		*Internal prestress*	
				Quantity	*Sum*	*Quantity*	*Sum*
Prestress	*Internal*	*tonne*	*2700*	*0*	*0*	*28.0*	*75,600*
	External	*tonne*	*3900*	*18.0*	*70,200*	*0*	*0*
Concrete	*50/20*	*m^3*	*55*	*409*	*22,495*	*456*	*25,080*
Reinforcement	*>=20mm*	*tonne*	*550*	*46.5*	*25,575*	*20.5*	*11,275*
	<20mm	*tonne*	*750*	*30.8*	*23,100*	*33.8*	*25,350*
Formwork	*Internal*	*m^2*	*30*	*912*	*27,360*	*850*	*25,500*
	External	*m^2*	*35*	*991*	*34,685*	*991*	*34,685*
Total (Superstructure excluding furniture etc)					*203,415*		*197,490*

The rates used were obtained from prestress manufacturers assuming they had a supply, install and stress contract. It is not clear how realistic the comparison is. However, if anything, it is likely to be slightly biased in favour of the conventional system as, although an allowance has been made for fixing the conventional ducts, the indirect effects of this (such as on programme) which are often quoted as a major reason for adopting external prestress, are not.

The externally prestressed design requires significantly more reinforcement than the conventional design and there are several quite distinct reasons for this. The externally prestressed bridge was designed as partially prestressed and additional unstressed reinforcement was required at critical sections to make up the shortfall of ultimate strength. The deviators in the externally reinforced structure require reinforcement. The anchorage blocks also require significantly more reinforcement than with internal prestress due to the fact that they have to span between the flanges and webs whereas in the conventional design they are directly supported by the webs. Finally, due to the use of the reinforced concrete column approach for shear in BD 58, significantly more links are needed at the critical sections. Unlike the other differences noted, which would still apply if Eurocode 2 were used, this difference would not apply in a design to the Eurocode. The higher prestress force, greater inclination of the tendons at the critical sections due to the curvature, and the thicker webs all have the effect of increasing the difference in the theoretical link requirement between the two designs. However, the difference is not fully reflected in the overall difference in the weight of the links. This is partly because the thicker webs increase the minimum nominal link requirement so that the mid-span link requirement in the internally stressed design is actually greater than with external prestress. However, a more important reason is that the simple single links used in the externally prestressed design are not practical in the internally prestressed design as they leave nothing to which the prestressing ducts can be fixed, the central ducts in particular.

The cost and quantity comparison of the two types of prestress is not by any means universally applicable and is greatly affected both by the choice of design and the detail of the code of practice used. If segmental construction is used, both types would require design for no-tension. This would have the effect that the prestress force required at the critical section would be greater with external prestress than with internal due to the reduced eccentricity. This difference would not, however, be fully reflected in the jacking force or, consequently, prestress quantity required due to the much lower friction losses with external prestress.

The prestress quantity in the conventional design could have been reduced significantly by stopping off some of the cables. Conversely, the reinforcement quantity in the external design could have been reduced significantly if a non-linear analysis were undertaken to determine the actual force in the tendons at the ultimate limit state. The difference in prestress force would have been less in a longer span design as the minimum external prestress would have been restricted by the requirement to maintain permanent compression. However, in a longer bridge, the advantage of low friction losses with external prestress becomes greater. Taking advantage of this, longer cables would normally be used with external prestress giving a significant saving in anchorage costs. Overall, the results are reasonably representative of an in-situ design to current UK standards although it appears the economics of the externally prestressed bridge would be improved if it were longer. This must be so as external prestress has been adopted for purely economic first cost reasons on many bridges, even when the design criteria were less favourable to it than those considered here.

The price difference is not really significant in relation to the realistic accuracy of the calculation. It should also be noted that, although the price quoted for internal prestress assumes this is to the new Concrete Society Specification (Concrete Society, 1996), the cost of the grouting trial this requires is not included. If it was, the internal prestressed design would become more expensive than the external one. However, this would make the comparison unrepresentative as normally, either because of a bigger bridge or because of other bridges on the same contract, the cost could be spread over a greater quantity of prestress.

The biggest change to the relative quantities would arise if, as in some countries (such as Switzerland), the internally prestressed design were also allowed to be designed as partially prestressed. The prestress could then be reduced significantly. Because internal prestress is used more efficiently at the ultimate limit state, the reinforcement required to make the section work at the ultimate limit state would be lighter than with external prestress.

7.2 Whole life costing

Both internal and externally prestressed bridges, including their tendons, are designed for a nominal 120 year life. The prestressing systems are not intended to require any significant maintenance in that period. It is likely that the expansion joints and possibly bearings would require replacing but these are essentially the same for the two designs and were not included in the costs and quantities in Table 1.

The only routine requirement for the part of the structure included in the cost comparison is for inspection. The cost of a normal General or Principal Inspection for the two bridges are very similar. The additional cost for Phase 3 of a Special Inspection for the internally prestressed bridge would be approximately £10,000 at 1996 prices assuming Phases 1 and 2 did not reveal any abnormal evidence of problems which required more than normal further investigation. If this were done at 15 year intervals, the discounted present cost, based on the Treasury 8% discount rate, would be £4,000. This excludes traffic delay costs which may be involved if tendons had to be accessed from above or if the outer row of tendons had to be accessed from below. The significance of the latter obviously depending on what the bridge crossed.

There is no standard established equivalent of a Special Inspection for an externally prestressed bridge. However, the prestressing system is designed to be removable and re-stressable. Taking advantage of this, it may be considered desirable to remove and inspect some strands, or at least to carry out lift off tests to check the prestress force. This should be cheaper than a full Special Inspection of the internally prestressed bridge. It would also give a more reliable indication of the integrity of the prestress than conventional tests on an internally prestressed bridge which involve stress relief techniques. It could also be done with no disruption to normal traffic. Assuming both structures had satisfactory durability, the saving of traffic delay cost relative to the conventional design is the only difference which would be really significant, after discounting, in relation to the initial cost.

If, in contrast, it is assumed that the prestress required replacing relatively early in the life of the bridges, the difference between the whole life costs would be dramatic, even without including traffic delay cost. Replacing internal prestress is not practical and it would probably be necessary to demolish and replace the bridge. The alternative would be to provide additional prestress, which would have to be external. This is a major operation requiring new anchorages and deviators to be installed. Replacing the prestress in the externally prestressed structure would be straightforward. However, due to the discount rate used, the present value of the cost of replacing the bridge later in its life would be much less significant: after 30 years it would be only some 10% of the initial cost.

It is possible to speculate as to other things which may alter the whole life cost. If, some years after construction, it was decided to strengthen the bridges for an increase of some 15% in live load, this could be accommodated in the externally prestressed bridge by simply re-stressing the cables to recover the long term losses in contrast to quite major strengthening required of the conventional structure.

Overall for this particular structure, there is a slight initial cost penalty for using external prestress but this could be considered justified by the advantages.

8 Conclusions

A design has been produced for an externally prestressed bridge which is representative of likely practice conforming to BD 58. This report contains a description of the main features of the bridge and indicates how the concept design was devised at. It describes the principles involved in the analysis appropriate for this type of construction and how these differ from the analysis of conventionally post-tensioned bridges. Detailed calculations for shear and flexural calculations are included in the report. It is intended that this report be used in conjunction with BD 58 for the design of externally post-tensioned bridges.

An equivalent design for a conventional internally prestressed bridge has also been produced for comparison. Simple costing suggests that, for this particular case, the conventional design is marginally cheaper in terms of first cost. These saving can be considerably out-weighed when whole life costs are considered due to the accessibility of the tendons for inspection and replacement.

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Appendix A: Hand calculation of bending moment capacity

Consider the critical section in sagging.

Global analysis was carried out using an uncracked elastic line beam analysis with the prestress moment considered on the load side of the equation. The required ultimate moment capacity is therefore that about the elastic neutral axis with the prestress force considered as being applied at the elastic neutral axis.

Steel forces are:

Tendon force = 29.15MN (after all losses and with γ_{fL} of 0.87 applied)

Bottom face steel is alternate T32 and T40 at 125mm and flange width is 5.2m, thus:

As = 43300 mm^2

Force = 43300x460/1.15 Assuming steel yields fully

= 17.32MN

Also use steel on top face of bottom flange which is T20 at 250mm,

= 6600mm^2

Force = 6600x460/1.15

= 2.64MN Again assuming steel yields fully

Total force to be resisted by concrete is

= 29.15 + 17.32 + 2.64

= 49.11MN

Use the simple rectangular concrete stress block.

If the neutral axis is in the top flange, the depth to it, d

= 49.11/(0.4x60x9.2)

= 0.222m

Thus the neutral axis is in the flange. BS 5400 allows the rectangular stress block to be used and the strain in the reinforcement considered clearly is sufficient for full yield.

Take moments about the elastic neutral axis which is 0.712m from the top.

Ultimate moment = 49.11 (0.712 - 0.222/2)
+ 17.32 capacity (1.8 - 0.712 -0.071)
+ 2.64 (1.8 - 0.712 - 0.25 + 0.057)
= 49.5MNm

The capacity calculated by SAM was 53MNm. The main reason for the difference is that the nominal steel in the other faces, ie, both faces of the webs and also the top and bottom faces of the top flange, was included in the SAM analysis. This could be done by hand but much of the steel is too close to the neutral axis to yield fully. The strain and force in each set of bars would therefore have to be calculated individually from the estimated neutral axis position.

The capacity of the section over the support would be calculated in the same way. However, the neutral axis is further from the compression face and not in the flange. This means that strictly the full rectangular-parabolic stress block for the concrete should be used. Also, the reinforcement does not fully yield. The solution therefore becomes iterative: it is necessary to assume or estimate a neutral axis position and then check that the forces match. If they do not, the neutral axis position is adjusted and the force calculations repeated. Once the forces match, the correct neutral axis position has been used and the moment capacity is calculated as above.

Doing a hand section analysis like this is not difficult. It is, however, only possible to check an assumed section works: the reinforcement cannot be designed directly for a required capacity. The design process therefore becomes iterative with many section analyses required. It is therefore invariably worth using a computer program.

Abstract

There has been increasing interest in recent years in the design and construction of post-tensioned concrete bridges with external unbonded tendons. To assist in the design of these structures, the Highways Agency issued a Standard and Advice Note (BD 58 and BA 58) in 1994. These documents were based on a review of existing knowledge that included recommendations on where further research was required.

To provide further information for designers an outline design for a continuous bridge with an overall length of 80m is presented in this report. The objective is to illustrate the concept and to present sample calculations. The bridge considered is a box girder structure, representing a highway bridge with two spans, 32m and 48m, carrying a two-lane carriageway over a motorway. The report contains a description of the main features of the bridge and describes how the concept design was devised. It describes the principles involved in the analysis appropriate for this type of construction and how these differ from the analysis of conventionally post-tensioned bridges. Detailed calculations for shear and flexural calculations are included in the report.

An outline design for an equivalent internally prestressed bridge is also given and costs are presented for both structures. The internally prestressed bridge is found to be marginally cheaper when initial costs only are considered. However, other factors could be considered to outweigh this. These include the costs of maintenance, inspection and potential strengthening, as well as traffic delay costs associated with these activities.

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